Modal Analysis of Quill Shaft in Turbo-generator Unit Based on SolidWorks

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ABSTRACT

The main goal of this work is to carry out a numerical modal analysis of a Quill shaft of turbo-generator unit affiliate to Mosul gas turbine station, using a trail version of popular finite element analysis software SolidWorks. The main function of Quill shaft is to protect the turbo-generator unit against overloads due to electrical network faults. The high flexibility of this shaft makes it capable of absorbing high displacements of resonance phenomena. This analysis is essentially needed to study the effect of transient loads applied to Quill shaft of turbo-generator unit under severe loading conditions such as electric network disturbances. The first five values of critical frequencies and mode shapes of axial, bending, and torsional vibrations were studied and analyzed. Each mode has been isolated separately by applying a special type of boundary conditions (restraints) available in program. The three types of natural frequencies have been found and reported. It was observed that the fundamental values of each three types of natural frequencies are relatively high and out of the range of Quill shaft operating speed. Finally, it has been concluded from all analyses that Quill shaft under consideration is safe from the stand point of modal analysis. The results show that the Quill shaft is not running at any of each three types critical speeds. Therefore, the resonance phenomenon for all three types of vibrations can not be happened no matter how high the amount of transient load applied.

Keywords:
SolidWorks; Finite element method; Turbo-generator unit; Natural frequency; Modal analysis; Quill shaft; Simulation.

1. INTRODUCTION

S. M. J. Ali, Z. Y. Mohammad and F. Q. Yahya [1] developed a model, Fig.1, for actual turbo-generator unit of Mosul gas turbine station and governing equations of this model has been formulated to estimate their respective parameters. The turbo-generator unit shown in Fig.1, is exposed to high axial, bending, and torsional oscillations due to high disturbances in the electrical grid during the connection of the generator unit to that grid. The main reasons of these faults are: the short-circuit, open-circuit, and sudden load changes. In addition, the outdoor parts of the electrical system (overhead lines, electrical towers, circuit breakers, etc.) experienced to different types of severe atmospheric conditions like ice, winds, moisture, etc., may play an important role in causing these types of faults.
Qing He and Dongmei Du [2] showed that the coupling interaction between electrical network faults and torsional vibrations makesshaft of turbine-generator unitvibrate. Therefore, alternating torsional stresses due to resonant torsional vibrations. Also, when any type of faults occurs in the grid, the electromagnetic torque between stator and rotor changes rapidly. As a result, torsional vibrations of shaft occur, alternate shear stresses due to these vibrations decrease the shaft life, and may be break the shaft.

For this reason, it becomes important to compute and analyze the natural or resonate frequencies and their corresponding mode shapes of all three types of vibrations (axial, bending, and torsional) agitated by the faults of electrical network, in order to obviate and stop disastrous accidents. The present work is related to Mosul gas turbine station which contains multi turbo-generator units for producing electrical power. The most important and critical part of the turbo-generator unit is the Quill shaft. The purpose of Quill shaft is to connect the gearbox reducer with the generator as shown in Fig. 2. Its main function is to work as a mechanical fuse for the unit to protect it against overloads and electrical faults and disturbances. The high axial, bending, and torsional flexibility owned by Quill shaft material make it deformed or brake before any other part of the unit because of its high capacity to absorb the induced large axial, bending, and torsional displacements.

The main goal of the present work is to find the values of resonate natural frequencies of the Quill shaft for three conditions: axial, bending and torsional and its corresponding mode shapes, by using numerical method (Finite element method) and compare them with the forced frequency (actual operating speed of the shaft). If one or more of these natural frequencies is located within the value of operated speed of the shaft, then, it is said that the shaft is in resonance condition, the case that should be avoided in the initial or early design stages. However, in this case, a redesign or improved design of Quill shaft should be introduced. This step may be carried out in the future work.

2. MODAL ANALYSIS THEORY

2.1 Basic theory of modal analysis

Yichang Liu, Chunlin Tian, Tao Liu, and Fanwu Kong [3] defined the modal analysis as a numerical technique to calculate the characteristics of structure. The vibration characteristics of any structure including critical frequency and mode shape.

Li Cheng and Hongguang Wang [4] stated that the critical frequency and mode shape considered the two significant factors on the dynamic properties of any mechanical structure, which are played an important role in the design of the structural system for vibration application. Both of them depend on the mass and stiffness properties of the structure. In addition, the damping has low effect on the critical frequency and mode shape, therefore, the damping coefficient can be cancelled in the modal analysis.

For a linear system of free vibrations, the differential equations can be summarized in a matrix form [5].

\[ \mathbf{M} \ddot{\mathbf{X}} + \mathbf{C} \dot{\mathbf{X}} + \mathbf{K} \mathbf{X} = \mathbf{0} \] ………………………..(1)

Where

\( \mathbf{M} \) is the mass matrix \((n \times n)\)

\( \mathbf{K} \) is the stiffness matrix \((n \times n)\)

\( \mathbf{X} \) is the n-dimensional column vector of generalized coordinates.

The general form of the differential equations for a free vibrations of a linear undamped n-degree of freedom system is

\[ \mathbf{M} \ddot{\mathbf{X}} + \mathbf{K} \mathbf{X} = \mathbf{0} \] ………………………..(2)

Objectives of the present work

The objectives of the present work is to find the values of resonate natural frequencies of the Quill shaft for three conditions: axial, bending and torsional and its corresponding mode shapes, by using numerical method (Finite element method) and compare them with the forced frequency (actual operating speed of the shaft). If one or more of these natural frequencies is located within the value of operated speed of the shaft, then, it is said that the shaft is in resonance condition, the case that should be avoided in the initial or early design stages. However, in this case, a redesign or improved design of Quill shaft should be introduced. This step may be carried out in the future work.
The solution of equation (2) can be given in the form

\[ \mathbf{x}(t) = \mathbf{x}e^{\omega t} \]  \hspace{1cm} (3)

Where \( \omega \) is the vibration frequency and \( \mathbf{x} \) is an \( n \)-dimensional vector of mode shape.

The values of \( \omega \) such that equation (3) is a solution of equation (2) are called the natural frequencies. Each natural frequency has at least one corresponding mode shape.

Substitution of Equation (3) into Equation (2) leads to

\[ (-\omega^2 \mathbf{M} \mathbf{x} + \mathbf{K} \mathbf{x})e^{\omega t} = 0 \]  \hspace{1cm} (4)

Since \( e^{\omega t} \neq 0 \), for each real value of \( t \),

\[ -\omega^2 \mathbf{M} \mathbf{x} + \mathbf{K} \mathbf{x} = 0 \]  \hspace{1cm} (5)

The mass matrix \( \mathbf{M} \) is nonsingular, and thus \( \mathbf{M}^{-1} \) exists. Multiplying equation (5) by \( \mathbf{M}^{-1} \) and rearranging gives

\[ (\mathbf{M}^{-1} \mathbf{K} - \omega^2 \mathbf{I}) \mathbf{x} = 0 \]  \hspace{1cm} (6)

Where \( \mathbf{I} \) is the identity matrix (\( n \times n \)).

Equation (6) is the matrix form of a system of \( n \)-simultaneous linear algebraic equations. Applying Cramer’s rule to obtain the solution for the \( j \)th component of \( \mathbf{x} \), \( X_j \)

\[ X_j = \frac{0}{\det[\mathbf{M}^{-1} \mathbf{K} - \omega^2 \mathbf{I}]} \]  \hspace{1cm} (7)

Thus the solution \( (\mathbf{x} = 0) \) is obtained unless

\[ \det[\mathbf{M}^{-1} \mathbf{K} - \omega^2 \mathbf{I}] = 0 \]  \hspace{1cm} (8)

Hence, \( \omega^2 \) must be an eigenvalue of \( \mathbf{M}^{-1} \mathbf{K} \). The mode shape is the corresponding eigenvector. Natural frequencies of multi-degree of freedom systems are computed as the quadratic roots of the eigenvalues of \( \mathbf{M}^{-1} \mathbf{K} \), i.e.,

\[ \omega_n = \sqrt{\frac{\mathbf{K}}{\mathbf{M}}} \]  \hspace{1cm} (9)

Similarly, the equation of the angular motion can be obtained by the same way [6]:

\[ J\ddot{\theta} + \mathbf{K}_t \theta = 0 \]  \hspace{1cm} (10)

which can be seen to be comparable to equation (2) \( i \mathbf{J} \) and \( \mathbf{K}_t \) are the \( n \) \( x \) \( n \) polar mass moment of inertia and torsional stiffness matrices respectively, and \( \theta \) is the \( n \)-dimensional column vector of generalized coordinates. Thus the natural circular frequency of the torsional system is

\[ \omega_n = \frac{\mathbf{K}_t}{\sqrt{\mathbf{J}}} \]  \hspace{1cm} (11)

### 2.2 Steps of the FEM applying to problems of vibration

The FEM is a numerical technique that can be used for an accurate but approximate solution of many complex mechanical and structural vibration problems [6, 7].

In this method, the actual structure is replaced by several pieces or elements, each of which is assumed to behave as a continuous structural member called a finite element. The elements are assumed to be interconnected at joints or nodes. The mass and stiffness matrices and force vectors needed for the finite element analysis can be derived for the basic one-dimensional elements such as a bar in axial motion, a rod in torsional motion, and a beam in bending motion. The boundary conditions should be incorporated to the conditions to the assembled system matrices and equations. For clarification, an example was presented here of uniform bar considered as one-dimensional element, the two end points form the joints or nodes as shown in Figure below [8].

The element is subjected to axial loads \( f_i(t) \) and \( f_j(t) \), the axial displacement \( u \) within the element is linear in \( x \). The joint displacements \( u_1(t) \) and \( u_2(t) \) are unknown. After several steps of derivation, the mass and stiffness matrices along with force vector can be identified as:

\[ [\mathbf{M}] = \frac{\rho A l^2}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \]  \hspace{1cm} (12)
\[ [K] = \frac{EA}{l} \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \] ...................................(13)

And the force vector as:
\[ \vec{f} = \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix} \] ...................................(14)

Where ρ, E are density and modulus of elasticity of the bar material respectively, A is the cross-sectional area, and l is bar length.

Most softwares using finite element techniques which included frequency analysis gives the results of all types of natural frequencies (axial, bending, and torsional) in a confusing or mixed way and not separately. This makes the sorting or isolating process of each type of natural frequencies is difficult, tedious, and consumes more time.

To facilitate this work and reduce the effort as well as the execution time of program running, a control on the selection of the type of constraints or fixtures used in the model is applied so that each of three types of natural frequencies can be found individually. Figs. 3, 4, and 5 show the types of fixtures (boundary conditions) used for axial, bending, and torsional frequencies analysis in SolidWorks.

3. FINITE ELEMENT METHODOLOGY

3.1. Finite element method

FEM simulates a solid part by splitting the its geometry into a number of standard shape elements, applying loads and restraints, assign material, then computing variables of interest like deflection, stresses, …etc. The behavior of each element is depicted by a set of equations. Just as the set of elements would be joined individuals elements are joined into a set of equations that depict the behaviors of the whole structure. Any continuous body has finite degree of freedom and its not possible to solve in this format. With the help of discretization, i.e. nodes and elements, FEM reduces degree of freedom from infinite to finite. All computations are made at finite number of points known a particular shape such as quadrilateral or triangular ….etc, is known element. To obtain the value of variable say displacement between the calculation points, interpolation function is used. The solution of any engineering problem is carried out by three methods: analytical method or hand calculations, numerical method, and experimental method or physical testing. FEM is classified under numerical method denomination. The program takes the elements which have been defined, puts the equations for each unknown value, and collects them together as equation of matrix form. Finally, solve all these for the unknown parameters

<table>
<thead>
<tr>
<th>Alloy Steel</th>
</tr>
</thead>
</table>
| 1 Yield Strength (Sy) | 620.422 MPa  
| 2 Tensile Strength (Su) | 723.825 MPa  
| 3 Elastic Modulus (E) | 210 GPa  
| 4 Shear Modulus (G) | 79 GPa  
| 5 Poisson’s Ratio (ν) | 0.28  
| 6 Density (ρ) | 7700 Kg/m³ |

[9].

3.2 Finite element modeling and simulation
To construct the Quill shaft model using FEM method, a trial version of SolidWorks 2011 has been used. The following steps give the modeling and simulation for the modal frequency analysis carried out by SolidWorks:

1. **Model Building**
   Building a solid (3D) model of the Quill shaft using the actual dimensions extracted from Mosul gas turbine station. The solid (CAD) model and detailed dimensions of the shaft are shown in Figs. 6 and 7 respectively.

2. **Material Selection**
   Assign the properties of the actual material from which the Quill shaft is manufactured. These properties are given in Table 1.

3. **Fixtures (boundary conditions) definition**
   Refer to Figs. 3, 4, and 5 again here, show the three types of fixtures used in the present modal analysis to isolate axial, bending, and torsional frequencies of Quill Shaft. A constraint means exactly zero displacement in the direction required.

4. **Model Meshing**
   Results accuracy and speed of calculations are influenced directly by the type of mesh or grid used. Therefore, meshing is considered as the critical step in finite element analysis. Meshing of a geometric object represents a set of finite elements used for computational analysis or modeling. As the number of elements increased, the accuracy of results increased and the speed of calculations decreased. As a compromised solution, medium mesh rather than coarse or fine mesh has been used to obtain the accepted results, considering the computer operation ability. The type of element is solid have a minimum size of 17.32 mm and a maximum size of 86.632 mm has been adopted. The mesh model of Quill shaft is shown in Fig. 8.

5. **Solution Form Selection**
   FFEPlus is selected as the solution form as available in SolidWorks.

6. **Natural Frequency Determination**
   The first 5th natural frequencies have been calculated for each type (axial, bending, and torsion). The 1st natural frequency is always considered the fundamental or lower value which represents the closest value to the operated speed of the Quill shaft.

7. **Final Results**
   The eventual results of analysis can be acquired by running the program including critical speeds and inherent mode shapes for three types of vibrations.
Modal Analysis of Quill Shaft

a) First mode (277.48 Hz)
b) Second mode (656.27 Hz)
c) Third mode (1231.4 Hz)
d) Fourth mode (2019.7 Hz)
e) Fifth mode (2681.3 Hz)

Fig. 9: The first five modes of bending natural frequency.
Fig. 10 The first five modes of axial natural frequency
Fig. 11 The first five modes of torsional natural frequency

a) First mode (654.8 Hz)
b) Second mode (1533 Hz)
c) Third mode (2186.6 Hz)
d) Fourth mode (3532.3 Hz)
e) Fifth mode (4592.7 Hz)
4. RESULTS
The vibration characteristics of Quill shaft in turbo-generator unit was obtained with modal analysis. The first five values of mode shapes of Quill shaft was found with their bending, axial and torsional natural frequencies and the deformation distribution. Each type of natural frequency has been separated in order to characterize each mode accurately and obviously. Table 2 show the bending, axial and torsional natural frequencies of the first five modes of the Quill shaft, and the inherent mode shapes is displayed in Figs. 9, 10, and 11 respectively.

The results show that the values of natural frequency seems to be increasing from first modeshape to last mode shape for all three types as shown in Fig.12 in terms of cycle/sec (Hz), and Fig.13 in terms of revolution/min (rpm). Also, it is observed that the values of resonant natural frequencies are out of the rotating speed of Quill shaft in each type. The interval between each natural frequency mode is comparatively high for all three types as shown in Table 2.

From the mode shapes shown in Figs. 9, 10, and 11, along with Table 3 and Fig.14 for maximum values of resultant displacements, it can be seen that the maximum amplitudes of shaft deformation or displacement occur at all modes for torsional natural frequencies, and the minimum amplitudes occur at second mode for axial natural frequency.

Also, it can be observed from Figs.9,10,and 11, or table 2, that the minimum value of resonant natural frequency which represents the nearest value to Quill shaft operating speed (3000 rpm=60 Hz) is the first mode of bending frequency (16648.8 rpm=85.66 Hz).

All types of natural frequencies (bending, axial, and torsional) are relatively high (out of the range of rotating speed of shaft, six times minimum for bending, 13.1 times medium for torsional, and 21.63 times high for axial) considered as safety margins comparing with fundamental natural frequency as reported in Tables 4, 5, and 6.

5. CONCLUSIONS
The solid model of mechanical structure of the Quill shaft was inserted in this work. The modal analysis using FEM with SolidWorks simulation is offered. The three types of critical frequencies

![Fig. 12 Natural frequencies in (Hz)](image)

![Fig. 13 Natural frequencies in (rpm)](image)

Table 2: Results of natural frequencies of the first five mode shapes

<table>
<thead>
<tr>
<th>Mode</th>
<th>Natural Frequencies, Hz(rpm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bending</td>
</tr>
<tr>
<td>1st</td>
<td>277.48</td>
</tr>
<tr>
<td></td>
<td>(16648.8)</td>
</tr>
<tr>
<td>2nd</td>
<td>656.27</td>
</tr>
<tr>
<td></td>
<td>(39376.2)</td>
</tr>
<tr>
<td>3rd</td>
<td>1231.4</td>
</tr>
<tr>
<td></td>
<td>(73884)</td>
</tr>
<tr>
<td>4th</td>
<td>2019.7</td>
</tr>
<tr>
<td></td>
<td>(121182)</td>
</tr>
<tr>
<td>5th</td>
<td>2681.3</td>
</tr>
<tr>
<td></td>
<td>(160878)</td>
</tr>
</tbody>
</table>
almost six times the shaft operating speed, as shown in Table 4 for bending mode.

3) Forcing frequency of Quill shaft is 50 Hz, which is very lower than of any of each three types of critical speeds. Therefore, the resonant condition cannot be happened and Quill shaft is not running at critical speed.

4) However, practically, if the generator-unit is exposed suddenly to some types of severe loading conditions, such as open circuit or unloading fault, then the speed of Quill shaft will be increasing drastically to values much more than the operating speed and may be exceeded the resonant value. In this case, the redesign of Quill shaft should be carried out.

6. REFERENCES


تحليل السرع الحرجة واشكال الطور
لعمود الدوران الملتف التابع لوحدة التوليد التوربينية باستخدام برنامج SolidWorks

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الخلاصة

الهدف الرئيسي من هذا البحث هو إجراء تحليل عددي للسرع الحرجة (الذبذبات الطبيعية) واشكال الطور لعمود الدوران الملتف لوحدة التوليد التوربينية التابعة لمحلة الموصل الغازية باستخدام نسخة تجريبية لبرنامج SolidWorks. الهدف من هذا البحث هو حماية وحدة التوليد التوربينية من تأثيرات الاحمال العالية أو المفرطة والتي تنتج من الأعمدة الكهربائية المتكبدة في الشبكة، وذلك بسبب امتلاك عمود الدوران المرونة عالية تجعله قابل لامتصاص القمم العالية للتشوهات المحورية أو التغطية أو الأفلاط ظاهرة التواء أو ظاهرة الدوارية في نظام التمزج الاهتزازية. عمود الدوران الملتف يتأثر بظروف التشغيل المتغيرة مثل اضطرابات الشبكة الكهربائية. تم دراسة تحليل القمم الخمسة الأولى للسرع الحرجة (الذبذبات الطبيعية) واشكال الطور المطابقة لها. وتم أيضاً معالجة وتشخيص ومن ثم حساب وتدوين كل نوع على حدة وذلك تطبيق أنواع خاصة من القواعد المتاحة في البرنامج. لوحظ بشكل واضح أن القم الأول أو الابتدائي للسرع الحرجة وتم أنه من القواعد المتاحة في برنامج برنامج SolidWorks للسودان في مراحل التشغيل. استنتجت جميع النتائج التي أجريت على هذا النوع من عمود الدواران التي تحقق اهتزازات الاهتزازات الرنينية وكيفية تجنبها. من هذه البداية، فإن ظاهرة الاهتزاز الرنينية التي تعني تطابق السرع الحرجة لعمود الدوران مع سرعة الاهتزازات المذكورة في ثلاثة من الاهتزازات المذكورة، بعد جدًا بغض النظر عن قيمة الحمل الاهتزازية الذي يُعرض له عمود الدوران.

تحليط السرع الحرجة واشكال التطور
لعمود الدوران الملتف التابع لوحدة التوليد التوربينية باستخدام برنامج SolidWorks

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