A Comparative Study of FFT Based Frequency Estimation Using Different Interpolation Techniques

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ABSTRACT

Fast Fourier Transform (FFT) is a commonly used method in electronic support systems for frequency parameter estimation. If the frequency of the radar signal is not an exact multiple of the frequency resolution, the frequency of this signal will usually appear in an inter-line position when FFT is applied. To improve the accuracy of the estimated frequency, interpolation techniques are used to find the peak between two spectral lines. In this study, the frequency of the radar signal is estimated by employing three different interpolation techniques (Ding, Voglewede and Hanning window based interpolation) to the output obtained by applying N-point FFT to the intermediate frequency (IF) signal. In addition, unlike the literature, the behavior of signals contaminated with Laplace noise as well as Gaussian noise were analyzed with these three techniques and their performances were compared. From the analysis results, Ding and Voglewede techniques reduced error rate at all frequency. However, the Hanning window-based interpolation method improved the frequency accuracy values at 500MHz and 750MHz, but it increased the error at 250MHz and 1000MHz frequencies. The error rates of the estimated frequencies can be sorted from the lowest to the highest as follows: Ding, Voglewede and Hanning window based interpolation.

Keywords:
FFT, Frequency Estimation, Interpolation, Laplace Noise, Gauss Noise

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1. INTRODUCTION

Frequency estimation has substantial importance for various application areas [1-3]. Electronic warfare (EW) systems are also one of the areas where frequency estimation is very important [4]. Nowadays, wars are carried out in the world of electronic warfare. Therefore, a significant increase has been observed in studies on electronic warfare systems in recent years [5–7]. Electronic warfare systems can be divided into 3 main categories as Electronic Support (ES), Electronic Attack (EA), Electronic and Protection (EP). The Pulse Descriptor Word (PDW) parameters required for the identification of the threat radar are extracted by the electronic support systems. PDW includes 5 important parameters: Time of Arrival (TOA), Pulse Width (PW), Angle of Arrival (AOA), Pulse Amplitude (PA) and carrier frequency (RF) [8]. Accurate estimation of carrier frequency is essential for the PDW extraction and deinterleaving. In literature, there are different methods for the frequency estimation but FFT is the most widely used method because it requires less computation and is faster than the Discrete Fourier Transform (DFT) [9]. In the FFT method, frequency resolution is calculated by dividing the sampling frequency by the number of FFT points (N). To make more frequency estimation, N can be increased to improve frequency resolution but this requires computational cost. Since computational cost delays the frequency estimation time, it is not preferred in EW systems. Interpolation methods can be used to improve frequency resolution without increasing the number of FFT points. Recently, many studies have been carried out in the literature [10–14], within the scope of obtaining the most accurate frequency by applying interpolation methods. For example; Gasior et al [15], preferred Parabolic and Gaussian interpolation method and also they are applied to different windows in time domain suchs, Gaussian, Blackman, Blackman-Harris,
Nuttall, Blackman-Harris-Nuttall. According to simulation results, although both methods improve frequency, Gaussian interpolation method gives more accurate results than Parabolic. In addition, Gaussian and Parabolic interpolation methods performed the frequency estimation with the lowest error rate when the Gaussian window was applied. Candan [16], presented a new interpolation method derived from the Jacobsen method. The proposed method called “Jacobsen with bias correction” and the Parabolic, Quinn, Macleod, Jacobsen methods were compared in terms of bias and root mean square error (RMSE). When the FFT number is 8, parabolic interpolation has the poorest bias and Jacobsen is the least biased estimator. If N is large enough, at high SNR values Jacobsen and Jacobsen with bias correction methods have approximately the same performance. Author thinks that the proposed method can perform well for low or medium N values. Fang [17], proposed a new estimator and compared it with Jacobsen with bias correction [16] and Jacobsen, Quinn, Improved Quinn, Macleod interpolation techniques which is available in the literature. From the simulation results, the proposed one has better performance than others. Iglesias et al. [18] before the FFT calculation, applied zero padding in the time domain. Then FFT was calculated and Parabolic interpolation was applied to improve the frequency. Based on the simulation results, a more accurate frequency result was obtained compared to the frequency obtained at the output of FFT.

In some studies in the literature, authors made modifications or additions to acknowledged interpolation methods. For instance, Quinn made some additions to Quinn’s first estimator then Improved Quinn or Quinn’s second estimator was obtained [19]. Koç [20] compared Quinn's first and Quinn’s second estimator methods, the second method has a lower error rate than the first. Also, Minda et al. [21], modified Jain interpolation method and obtained Corrected Jain algorithm and in the test result obtained generally Corrected Jain has better performance than Jain method. In another study, Minda et al.[22] analyzed 5 interpolation methods and this study included 2 stages. As the first stage, Quinn, Jain, Jacobsen Voglewede and Ding methods were compared according to RMSE. From the simulation results, Jacobsen has the smallest RMSE. Quinn was the second successful method and others have approximately the same error rates. At the second stage, instead of Jain interpolation, the Corrected Jain method was used. As a result of analyzes carried out, although it is difficult to choose the most successful method directly because the error rate of different methods is low at different time intervals, Jacobsen and Quinn are the 2 most successful methods in general. Niranjan et al. [23] compared rectangular window based, Hanning window based and curve fitting (CFT) based interpolation methods. In addition, to observe the effect of time domain, Hanning window to CFT, digitized IF signal was multiplied by Hanning Window and FFT was calculated and curve fitting interpolation technique applied. From the presented simulation results, RMSE of each method can be sorted from lowest to the highest as follows; CFT with Hanning window, CFT, Hanning window based interpolation, Rectangular window based interpolation.

In the literature, Ding, Voglewede and Hanning window based interpolation methods have not been analyzed at different frequencies, different SNR levels and different noises. Therefore, in this study a detailed analysis was made using these methods. The rest of the work is as follows, in chapter 2, mentioned about theory of FFT and interpolation methods and their formulas are shared. In chapter 3, detailed simulations were made and simulation results were discussed. Lastly, summary of this study and information about future studies were presented.

2. MATERIAL AND METHODS

The general block diagram of FFT based receivers, which are usually preferred in electronic support systems, is shown in Fig. 1[24] RF signal coming from the antenna is downconverted in RF chain block and IF signal is obtained. Then, IF signal is digitized with an ADC that has a high sampling rate and FFT is applied to digitized IF signal and frequency is obtained.

Let us consider a digitized sinusoidal signal as sampled sequence,

\[ v[n] = V \exp \left( j2\pi f_0 \frac{n}{f_s} \right), \quad n = 0, \ldots, N - 1 \]  \hspace{1cm} (1)

\[ \Delta f = \frac{f_s}{N} \]  \hspace{1cm} (2)

where \( V \) is amplitude, \( f_0 \) is the frequency of the signal, \( f_s \) is sampling frequency, \( N \) number of FFT points and \( n \) is the index of the samples.

Frequency resolution, ratio of sampling frequency to number of FFT points can be shown by Eq. (2).
If the frequency of the IF signal is not an exact multiple of frequency resolution, an accurate frequency cannot be obtained. Since accurate estimation of frequency has huge importance for the PDW extraction and deinterleaving of threat radar, interpolation methods can be used to improve FFT result. In this study, the proposed frequency estimation block is shown in Fig. 2 and frequency spectrum of FFT is given in Fig. 3[10].

\[
V_k = \sum_{n=0}^{N-1} v[n] e^{-j(2\pi/N)nk}, \quad V_k = \text{Re} V_k + j \text{Im} V_k
\]

\[
\text{Re} V_k = \sum_{n=0}^{N-1} v[n] \cos \left(\frac{2\pi}{N} nk\right), \quad \text{Im} V_k = -\sum_{n=0}^{N-1} v[n] \sin \left(\frac{2\pi}{N} nk\right)
\]

\[
V_k = \text{Re} V_k + \text{Im} V_k
\]

\[
|V_k| = \sqrt{(\text{Re} V_k)^2 + (\text{Im} V_k)^2}
\]
After \(V_i\) and its neighbours are found and correction coefficient is calculated, then frequency is recalculated by using Eq. (7)[22]

\[f_c = (k + \delta)\Delta f\]  \hspace{1cm} (7)

Although, the purpose of all interpolation methods is to improve the FFT result, the difference between them is the correction coefficient calculation method and so their formulas. The methods can be divided into 2 groups according to whether they use one or two neighbours of the \(V_i\) and use the magnitude of the real or complex part of \(V_i\) when calculating correction coefficient. As can be seen from the Eqs. (8-10), because Ding and Voglewede used \(V_i\) and its two neighbours they are three points method, while Hanning window based interpolation method is two points method. In addition, all three methods use the magnitude of complex \(V_i\) instead of using only the real part. In the Equations (8-10), firstly correction coefficients are calculated and then similar to Eq. (7), frequency is recalculated by using the correction coefficients.

<table>
<thead>
<tr>
<th>Method</th>
<th>Formula</th>
<th>(f) Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ding Method [19]</td>
<td>[\delta = \frac{V_{[k+1]} - V_{[k-1]}}{V_{[k-1]} + V_{[k]} + V_{[k+1]}}, \quad f_{\text{ding}} = \frac{f_x}{N}(k + \delta)]</td>
<td>(f_{\text{ding}})</td>
</tr>
<tr>
<td>Voglewede Method [7]</td>
<td>[\delta = \frac{V_{[k+1]} - V_{[k-1]}}{2(V_{[k]} - V_{[k-1]} - V_{[k+1]}), \quad f_{\text{voglewede}} = \frac{f_x}{N}(k + \delta)]</td>
<td>(f_{\text{voglewede}})</td>
</tr>
<tr>
<td>Hanning window based interpolation [22]</td>
<td>[\delta = \frac{2V_{[k+1]} - V_{[k]}}{V_{[k]} + V_{[k+1]}}, \quad f_{\text{hanning}} = \frac{f_x}{N}(k + \delta)]</td>
<td>(f_{\text{hanning}})</td>
</tr>
</tbody>
</table>

3. SIMULATION RESULTS AND DISCUSSION

In this section; Ding, Voglewede and Hanning window based interpolation methods are compared in terms of accuracy, RMSE, a commonly used evaluation criteria that uses the Euclidean metric to show the distance between estimated and predicted values. RMSE calculated by using Eq. (11). In formula, \(\hat{a}\) is predicted and \(a\) is the original value, \(N\) is the number of samples.

\[RMSE = \sqrt{\frac{\sum_{i=1}^{N}(a(i) - \hat{a}(i))^2}{N}}\]  \hspace{1cm} (11)

Matlab 2019a used for experiments of this study. For the detailed analysis of interpolation methods, 4 different IF signal frequencies were used as 250MHz, 500MHz, 750MHz and 1000MHz. SNR value varied from 10dB to 25dB with the step of 5dB to check the accuracy of methods. For each SNR level and frequency, 100 Monte Carlo simulations were carried out.

When a RF signal comes to the radar receiver, in addition to its frequency, it also includes environmental noise. Since the type and thus distribution of noises can not be known beforehand, in this study, in addition to Gaussian noise, RF signal was contaminated by Laplace noise to analyze the effect of different noises to interpolation methods.

Laplace noise was preferred because it has a steeper distribution than Gaussian and their formulas were given in Eqs (12) and (13) [25]–[27].

\[f_{\text{Gauss}}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\]  \hspace{1cm} (12)

\[f_{\text{Laplace}}(x) = \frac{1}{2\lambda} e^{-\frac{|x-\mu|}{\lambda}}\]  \hspace{1cm} (13)

To compare the method more clearly, simulation results were presented at 4 different tables (Table 1, Table 2, Table 3, Table 4). When Table 1 evaluated, for 100MHz IF signal, Ding shows the best performance for all SNR levels and for both noises. Hanning window based interpolation method affected FFT result adversely and has error rate higher than FFT without interpolation.
According to Table 2, when IF signal frequency is 500MHz, Hanning window based interpolation has the smallest RMSE when compared with Ding and Voglewede. It can be clearly seen that Ding and Voglewede improved the result of FFT without interpolation, but Hanning window based interpolation has a lower error rate than them.

From Table 3, for 750MHz, it is seen that all methods have approximately the same RMSE values but Ding has better performance than others. Although, all of the three methods improved the FFT result without interpolation.

When Table 4 is examined, Hanning window based interpolation method effected FFT result adversely similar to 250MHz. The error rates of the estimated frequencies can be sorted from the
According to the experimental results (Table 1, Table 2, Table 3, Table 4) the following inferences can be made. Except 500MHz Ding shows the best performance at any SNR value and both noise models among the 3 interpolation methods. Although the Hanning window based interpolation has a higher error rate than other methods, that is, lower performance, it is more successful than other methods at 500MHz. The Voglewede method did not increase the error in the FFT result for any frequency and for any SNR value, and it mostly improved the result for each frequency value. All methods were not affected by the change in noise type, that is, the method that performed well in Gaussian noise also performed well in Laplace noise. Similar to the study in literature [4], the change in SNR level did not significantly affect the FFT result and therefore the result of the interpolation method.

4. CONCLUSION

Accurate estimation of threat radar frequency has huge importance for the PDW extraction, deinterleaving and identification of the threat radar. FFT is a commonly used method for frequency estimation. To obtain a more accurate frequency, some different interpolation methods are applied to output of FFT. In this study Ding, Voglewede and Hanning window based interpolation methods are used to improve FFT results. When a RF signal arrives at the antenna it doesn’t include only its own frequency, it also includes noise. Since the type and source of noise can not be known in advance, radar signals are contaminated by Laplace noise, in addition to Gauss noise. Also, to examine methods more clearly, SNR level and frequency bandwidth are changed. When all tables are examined, it was seen that the noise type being Laplace or Gauss did not affect the error rate much. Based on thee simulation results, Ding is the most succesful interpolation method as it has the lowest error rate and Voglewede is the second succesful method. From all the tables, Ding and Voglewede improved the FFT result at all frequencies (250MHz, 500MHz, 750MHz, 1000MHz) and SNR levels(10dB, 15 dB, 20dB, 25dB) for both types of noise(Gauss and Laplace). Although, Hanning window-based interpolation method improved the accuracy of FFT output for the frequency values of only 500MHz and 750MHz, it has increased the error rate in the FFT result at 250MHz and 1000MHz frequency values. When the table of 500MHz frequency value is examined, it is observed that the Hanning window-based method has the lowest error rate and therefore the highest performance. When all tables are examined, it was seen that the noise type being Laplace or Gauss did not affect the error rate much. In future studies, maybe a new method is proposed and these methods can be compared with the proposed interpolation method. Also, existing methods can be changed to obtain more accurate results like Quinn’s second estimator.

NOMENCLATURE

ADC - Analog Digital Converter
AOA - Angle of Arrival
FFT - Fast Fourier Transform
IF - Intermediate Frequency
PA - Pulse Amplitude
PDW - Pulse Descriptor Word
PW - Pulse Width
RMSE - Root mean square error
SNR - Signal to noise ratio
$f_s$ - Sampling frequency
$\Delta f$ - Frequency resolution
$k$ - Bin number
$V[k]$ - Magnitude of the kth bin
$v[n]$ - Sampled signal sequence
$N$ - Number of FFT point
$\delta$ - Correction coefficient

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Abstract: The fast Fourier transform (FFT) is a common method used in electronic systems to estimate frequency parameters. If the radar frequency is not an integer multiple of the sampling frequency, the FFT will usually show that the frequency is between two lines. To improve the accuracy of frequency estimation, various estimation techniques are used to find the frequency between two lines. In this study, the frequency of the radar signal is estimated using three different estimation techniques (Ding, Voglewede, and Hanning) by applying N-point FFT on the average frequency (IF). In addition, unlike the literature, the performance of the three techniques was analyzed for noise Laplace and Gaussian, and compared.

The results showed thatAccent and Voglewede techniques reduced the error rate at all frequencies. However, the Hanning window estimation method improved the accuracy at 500 MHz and 750 MHz, but the error rate increased at 250 MHz and 1000 MHz.

The error rates at all frequencies are as follows: Ding, Voglewede, Hanning window.