

Mathematical Modelling and PID Controller Implementation to Control Linear and Nonlinear Quarter Car Active Suspension

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ABSTRACT

In this work, linear and nonlinear designs of active suspension models are proposed to develop and improve quarter-car systems. To simplify stability assessment, a second-order system is proposed for both linear and nonlinear cases. The linear system consists of mass, spring, and damper components, while the nonlinear system includes the same components with additional nonlinear parts for stiffness and damper. Moreover, the state space of the linear and nonlinear is presented as a preparatory step before applying the analysis methods to validate the models. After that, the stability of linear and nonlinear systems is characterized using Matlab simulations to compare suspension performance parameters such as rise time (t_r), settling time (t_s), and peak overshoot (M_p). The simulation results of the linear system for each of t_r , t_s , M_p were 0.097612sec, 2.3 sec, and 0.3839 cm, respectively, while the results of the nonlinear system were 0.52237 sec, 20.16 sec, and 0.3064 cm, respectively. In addition, the results for linear and nonlinear systems indicate the need to improve ride comfort and road handling using PID controller design. Consequently, it is possible to reach a better compromise than is possible using pure elements, without a controller. Finally, the active suspension system for both linear and nonlinear systems is improved through the application of a PID controller, resulting in the following values for the linear system: $t_r = 0.10721$ sec, $t_s = 1.693$ sec, and $M_p = 0.3682$ cm. Similarly, the nonlinear system showed improved performance with $t_r = 0.259775$ sec, $t_s = 1.325$ sec, and $M_p = 0.0734$ cm.

Keywords:

Active Suspension Systems, Linear Systems, Nonlinear Systems, Quarter Car, PID control.

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1. INTRODUCTION

For many years, ride comfort and car safety, the suspension system have been important subject to study. The purpose of the suspension system is to design and support the weight of the driver, passengers and the car's structure. In addition to the damping vibrations that reach the body of the car, handling stability, performance, and comfort of an automobile's suspension system can all be enhanced with thoughtful design. Currently, passive suspension is the most used type, where once the

suspension parameters have been chosen, they cannot be changed [1]. The concept of active vehicle engineering gained significant interest in the the 1960s, particularly in the research of active and semi-active suspension systems since. A fundamental approach based on the linear model and traditional PID system was adopted to investigate the dynamic suspension system, along with optimal linear quadratic control [2]. In contrast, passive suspension systems consisted of fixed and unchangeable components, leading to the drawback

of transmitting excessive road vibrations to passengers. To maximize ride comfort, the passive elements such as dampers and springs had to be carefully chosen for a softer portion of the vehicle while compromising comfort in rougher sections. Additionally, suspension system specifications needed to be adjusted to accommodate changing road conditions. Consequently, several approaches were developed, including partially active and fully active suspension systems, to address these challenges [3-5].

To analyse this, a quarter-car model representing one-fourth of the vehicle suspension system was developed for the sake of simplicity. This system connected the wheel and the body, which is a crucial component for transferring force and torque between the two [6].

Vehicle suspensions are designed to provide adequate road holding and isolate the vehicle body from road irregularities, addressing the challenge of offering comfort to occupants. Handling analysis is also concerned with achieving good road holding, which refers to car's ability to accelerate, brake, and turn safely [7-11]. In order to reduce body acceleration and dynamic tire load while still functioning within the limitations of the suspension working space for a specific suspension parameter set; the design had two goals. Traditional passive suspension systems aim to balance handling and riding. While a highly damped suspension offers excellent handling, it can make passengers uncomfortable. Conversely, a low damped suspension compromises vehicle stability but improves ride comfort. The effective control policy of an active suspension system allows for a balance between comfort and stability [12].

PID is the most popular control technique in business, and it has been utilized to control many systems, as mentioned in references [13] and [14]. Its ease of use and relative simplicity contribute to its appeal, either intuitively or by utilizing one of the various tuning techniques [15, 16]. It is also well-liked because it effectively modifies controller system parameters like overshoot, rise time, and settling time [17]. It requires high loop gains, however, and is not resilient to parameter fluctuation [18, [19].

Recently, active suspension systems have received significant attention from researchers interested in enhancing the vehicle's stability and ride handling capabilities. In the field of active suspensions systems, various control techniques have been employed, including the linear quadratic regulator [20], adaptive sliding control [21], H_{∞} control [22], sliding mode control [23], fuzzy logic [24], preview control [25], optimal control [26:27], and neural network methods [28]. These control techniques have the potential to improve the performance of active suspension systems. However, they often require more complex mechanisms or a unique performance determination table, in addition to posing certain application challenges.

This paper addresses the challenges associated with both linear and nonlinear suspension systems, which have exhibited inadequate performance and stability according to previous studies. The primary focus of this work is to identify and address the specific issues found in quarter active suspension systems, while also proposing future methods aimed at resolving these problems.

This paper aims to demonstrate how a PID inner loop feedback control of the actuator force, when combined with an input from a road disturbance, can improve the stability of a nonlinear quarter-car active suspension system.

2. PID Controller

The closed-loop control system serves as the foundation for the operating principle of PID controller, where PID stands for Proportional (P), Integral (I), and Derivative (D). In proportional control (P), the output signal is generated by multiplying the current error signal by the gain (K_p). The integral term encompasses the sum of all instantaneous values of the signal from the start of counting until the end, represented by the integral sign. By adding the to the proportional term, the process moves faster toward the set point and eliminates the residual steady-state error associated with a proportional controller as shown in equ (1).

The derivative term (D) slows down the output rate controller, and its impact is most noticeable when the controller is close to its set-point. The PID

controller employed in the active suspension system is depicted in Figure 1 [29].

$$PID = K_p e(t) + K_i \int e(t) dt + K_d \frac{d}{dt} e(t) \quad (1)$$

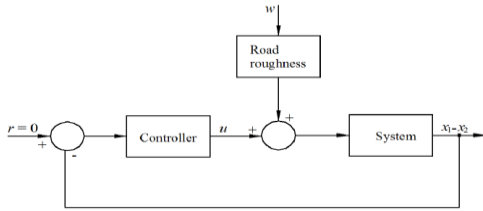


Figure 1. Block diagram of suspension system using PID Controller.

3. Linear Active Suspension System

In this section, the linear model of the 1/4 car active suspension system is presented, as shown in Figure 2. However, the linear model only consider the linear components of the dynamical nonlinear systems, thereby neglecting the nonlinearities in the stiffness and damper of the tire. In this model, an actuator generates the control force between the the wheels' mass and the vehicle body.

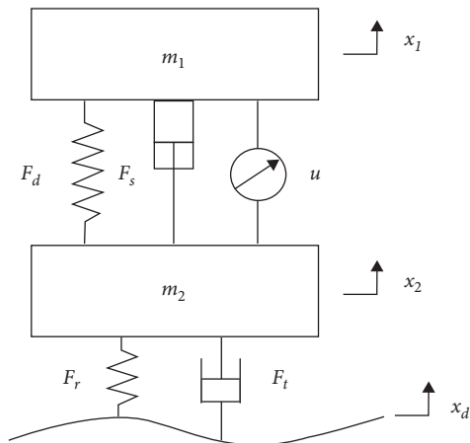


Figure 2. Model of quarter car active suspension system.

The variables used in this model include: m₁ for the sprung mass, m₂ for the unsprung mass, F_d for the spring elastic force, F_s for the damping force, F_r for the the tire elastic force, F_t for the the tire damping force, and u for the actuator control force. The following equations represent the motion dynamics of the car's body and wheels [31]:

$$m_1 \ddot{x}_1 + F_s + F_d + u = 0 \quad (2)$$

$$m_2 \ddot{x}_2 - F_s - F_d + F_t + F_r - u = 0 \quad (3)$$

Where F_d, F_s, F_r and F_t are:

$$\begin{aligned} F_d &= k_1(x_1 - x_2) \\ F_s &= c_1(\dot{x}_1 - \dot{x}_2) \\ F_r &= k_2(x_2 - x_d) \\ F_t &= c_2(\dot{x}_2 - \dot{x}_d) \end{aligned} \quad (4)$$

where x₁ and x₂ are represent the displacements of the sprung and unsprung mass, respectively, x_d represents the excitation displacement of the road, k₁ represents the linear stiffness coefficients of the spring, c₁ represents the linear damping coefficients of the suspension, while k₂ and c₂ represent the damping and stiffness coefficients of the tire, respectively.

$$m_1 \ddot{x}_1 + k_1(x_1 - x_2) + c_1(\dot{x}_1 - \dot{x}_2) + u = 0 \quad (5)$$

$$m_2 \ddot{x}_2 - k_1(x_1 - x_2) - c_1(\dot{x}_1 - \dot{x}_2) - k_2(x_d - x_2) - c_2(\dot{x}_d - \dot{x}_2) - u = 0 \quad (6)$$

3.1. MATLAB Simulation for Linear Suspension System

The simulation diagram of the open-loop system illustrates the interconnection of the linear active suspension systems, as shown in Figure 3:

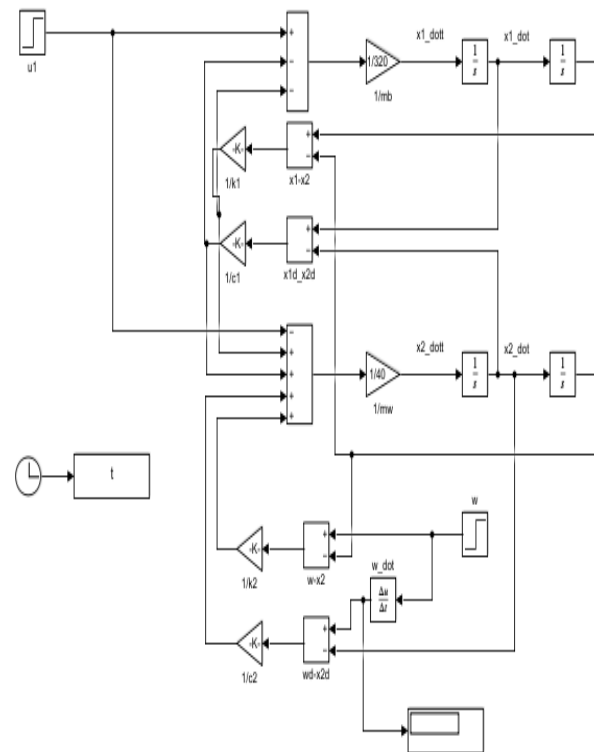


Figure 3. MATLAB-Simulink of 1/4 vehicle of the linear active suspension system.

4. Nonlinear Active Suspension System

In this section, a nonlinear two degree of freedom (2DOF) model is established for the design of a quarter car, taking into account the nonlinearity of the damping and elastic elements. The dynamic equations for the active suspension system of the quarter vehicle can be the same as equations(2) and (3), with F_d, F_s, F_r and F_t as defined [20]:

$$F_d = k_1(x_1 - x_2) + \alpha_1(x_1 - x_2)^3$$

$$F_s = c_1(\dot{x}_1 - \dot{x}_2) + \alpha_2(\dot{x}_1 - \dot{x}_2)^3 \quad (7)$$

where α_1 represents the nonlinear stiffness coefficients, α_2 represents the nonlinear damping coefficients of the suspension, while k_2 and c_2 represent the damping and stiffness coefficients of the tire, respectively. By substituting equations (7) and (4) into equations (2) and (3), we obtain the following [30]:

$$m_1\ddot{x}_1 + k_1(x_1 - x_2) + \alpha_1(x_1 - x_2)^3 + c_1(\dot{x}_1 - \dot{x}_2) + \alpha_2(\dot{x}_1 - \dot{x}_2)^3 + u = 0 \quad (8)$$

$$m_2\ddot{x}_2 - k_1(x_1 - x_2) - \alpha_1(x_1 - x_2)^3 - c_1(\dot{x}_1 - \dot{x}_2) - \alpha_2(\dot{x}_1 - \dot{x}_2)^3 + k_2(x_2 - x_d) + c_2(\dot{x}_2 - \dot{x}_d) - u = 0 \quad (9)$$

To simplify the model, the following parameter nominalization is made [20]:

$$\ddot{x}_1 + (x_1 - x_2) + \mu(x_1 - x_2)^3 + \zeta_1(\dot{x}_1 - \dot{x}_2) + \delta(\dot{x}_1 - \dot{x}_2)^3 + u = 0 \quad (10)$$

$$\ddot{x}_2 - \gamma(x_1 - x_2) - \gamma\mu(x_1 - x_2)^3 - \gamma\zeta_1(\dot{x}_1 - \dot{x}_2) - \gamma\delta(\dot{x}_1 - \dot{x}_2)^3 + \gamma k(x_2 - x_d) + \gamma\zeta_2(\dot{x}_2 - \dot{x}_d) - u = 0 \quad (11)$$

where $\mu = \frac{\alpha_1 L^2}{k_1}, \delta = \frac{\alpha_2 L^2}{\sqrt{m_1^3}}, \zeta_1 = \frac{c_1}{\sqrt{m_1 k_1}},$

$\zeta_2 = \frac{c_2}{\sqrt{m_1 k_1}}, k = \frac{k_2}{k_1}$ and $\gamma = \frac{m_1}{m_2}$. Here L is the unit length.

In the next section, the analysis of the linear and nonlinear systems is presented in order to test their stability and performance.

4.1. MATLAB Simulation for NonLinear Suspension System

The open-loop nonlinear system has been interconnected using MATLAB Simulink, as depicted in Figure 4

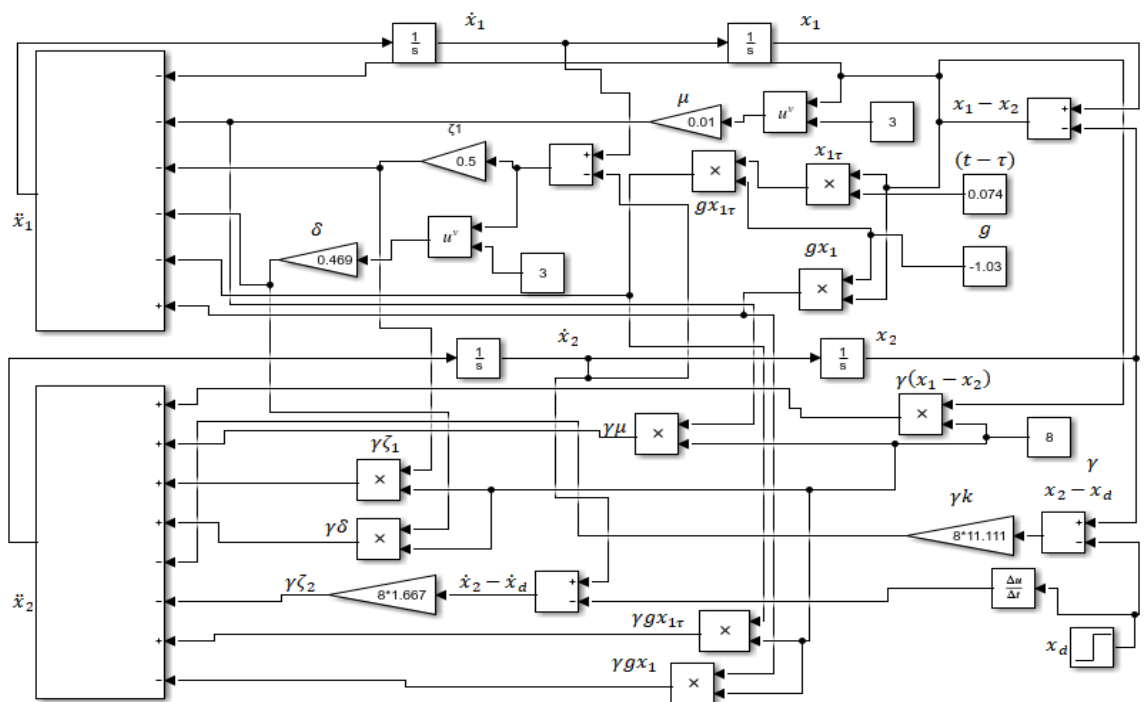


Figure 4. Matlab-Simulink diagram of the 1/4 vehicle with nonlinear active suspension.

5. RESULTS AND DISCUSSIONS

The results of the open loop for both linear and nonlinear systems display the responses and the analysis of the 1/4 quarter car active suspension systems. Table 1 presents the linear and nonlinear parameters of these systems.

Table 1. Parameters for linear and nonlinear systems [30, 31].

Parameters	Values	Units
m_1	320	kg
m_2	40	kg
c_1	1200	Ns/m
c_2	4000	Ns/m
α_1	180	N/m ³
α_2	20	N _s ³ /m ₃
k_1	18000	N/m
k_2	200000	N/m
μ	0.01	—
δ	0.469	—
ζ_1	0.5	—
ζ_2	1.667	—
k	11.111	—
γ	8	—

a) Linear system results

Figure 5 (a &b) illustrates the displacements of the body and wheel displacement respectively. The body displacement represents the state number one of the systems.

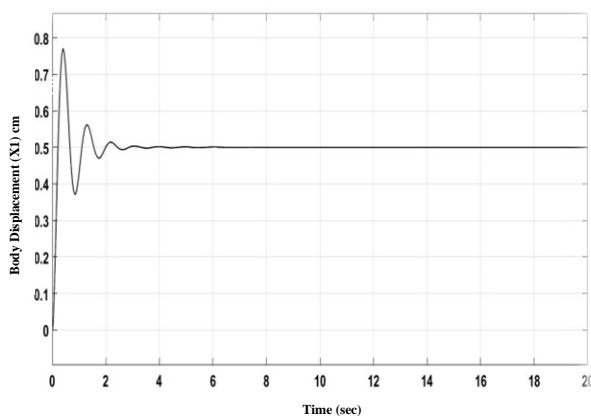


Figure 5 (a). The states of the linear active suspension system of Body displacement.

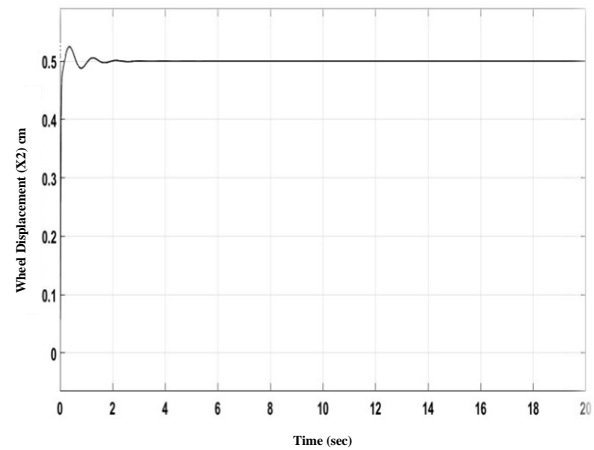


Figure 5 (b). The states of the linear active suspension system of wheel displacement.

Finally, the system output ($x_1 - x_2$) is presented in Figure 6, highlighting the necessity for enhancing both performance and stability. This improvement is accomplished through the utilization of a PID controller.

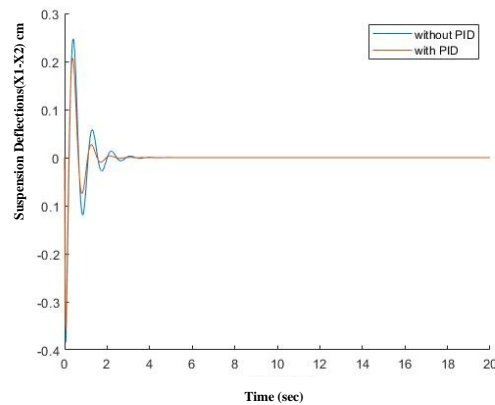


Figure 6. Comparison of output response ($x_1 - x_2$) in linear active suspension system with and without PID control.

b) Nonlinear system results

Figure 7 (a&b) illustrates the displacements of the body and wheel, respectively. The body displacement corresponds to the first state of the nonlinear systems.

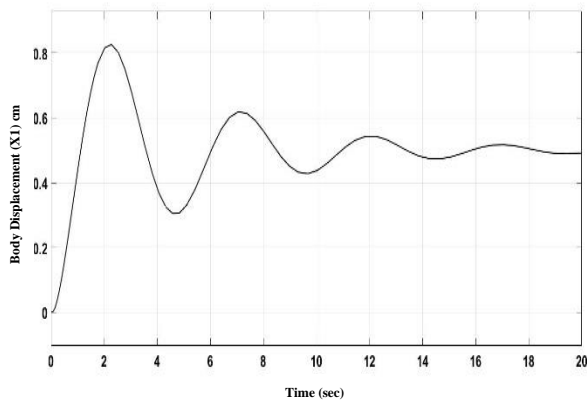


Figure 7 (a). Nonlinear active suspension system states for body displacement.

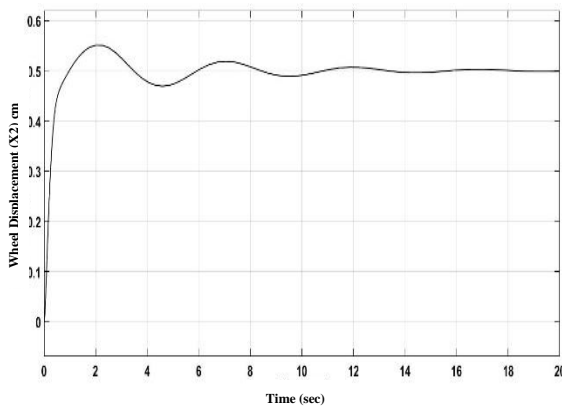


Figure 7 (b). Nonlinear active suspension system states for wheel displacement.

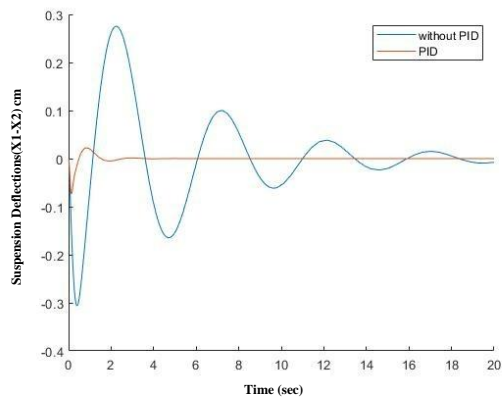


Figure 8. Output response ($x_1 - x_2$) of Nonlinear active suspension with and without PID control.

It can be argued that the mathematical models have been developed for both linear and nonlinear of active suspension systems. In addition, these systems are interconnected using Matlab Simulink,

as depicted in Figures 3 and 4, in preparation the next step. Subsequently, the systems are initialized with 0.5 road disturbance [20, 21]. The simulation results reveal that there is a need to enhance the stability and performance of these systems, and to address this, the implementation of a PID controller is suggested. Table 2 provides an overview of the characteristics of step input for both linear and nonlinear open-loop systems, with and without PID controller, including rise time, settling time, and maximum peak overshoot.

Table 2 systems characteristics for step input.

System	RiseTime (sec)	SettlingTime (sec)	Mp (cm)
Linear without controller	0.097612	2.3	0.3839
Linear with PID controller	0.10721	1.693	0.3682
Nonlinear without controller	0.52237	20.16	0.3064
Nonlinear with PID controller	0.259775	1.325	0.0734

6. Conclusion

The linear and nonlinear active suspension systems have been presented to address their respective issues. Firstly, the system model was designed and developed using linear and nonlinear concepts. Next, the systems were analyzed and tested to evaluate their stability. In the case of the system without a controller, the simulation results for the linear system were as follows: $t_r = 0.097612$ sec, $t_s = 2.3$ sec, $M_p = 0.3839$ cm. For the nonlinear system, the results were: $t_r = 0.52237$ sec, $t_s = 20.16$ sec, $M_p = 0.3064$ cm. Both the linear and nonlinear system responses indicated poor stability and performance. Finally, the active suspension system for both the linear and nonlinear systems was improved by implementing a PID controller. As a result, the performance of the linear system improved to $t_r = 0.10721$ sec, $t_s = 1.693$ sec, $M_p = 0.3682$ cm. Similarly, the nonlinear system showed improved performance with $t_r = 0.259775$ sec, $t_s = 1.325$ sec, $M_p = 0.0734$ cm.

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المخلص

في هذا العمل، يتم اقتراح تصميم خطي وغير خطي لنماذج التعليق النشطة من أجل تطوير وتحسين أنظمة ربع السيارة. وبالتالي، يقترح نظام الدرجة الثانية من أجل البساطة لتقييم استقرار النظام في كل من الحالات الخطية وغير الخطية. احتوى النظام الخطي على كتلة وزنبرك ومثبط بينما احتوى النظام غير الخطي على نفس المكونات مع إضافة أجزاء غير خطية للصلابة والمثبط. علاوة على ذلك، يتم تقديم مساحة الحالة الخطية وغير الخطية كخطوة إعداد قبل استخدام طرق التحليل للتحقق من صحة النماذج. بعد ذلك، يتميز استقرار الأنظمة الخطية وغير الخطية باستخدام محاكاة ماتلاب لمقارنة معاملات أداء التعليق مثل وقت الارتفاع (t_r)، وتسوية الوقت (t_s)، وذبذبة تجاوز (M_p)، وكانت نتائج المحاكاة للنظام الخطي لكل من t_r ، t_s ، M_p (0.097612 ثانية، 2.3 ثانية، 0.3839 سم) ونتائج النظام غير الخطية (0.52237 ثانية، 20.16 ثانية، 0.3064 سم) على التوالي. بالإضافة إلى ذلك، أظهرت نتائج الأنظمة الخطية وغير الخطية أن النظام يحتاج إلى تحسين راحة الركوب والتعامل مع الطريق بشكل أفضل باستخدام تصميم وحدة تحكم بييد، لذلك من الممكن الوصول إلى حل وسط أفضل مما هو ممكن باستخدام عناصر نقية (بدون وحدة تحكم)، وأخيراً، تم تحسين نظام التعليق النشط لكل من الأنظمة الخطية وغير الخطية من خلال تطبيق وحدة تحكم التكاملي التفاضلي التناسبي، وبالتالي تصبح النتيجة للنظام الخطي كانت لكل من t_r ، t_s ، M_p (0.10721 ثانية، 1.693 ثانية، 0.3682 سم)، نظام غير خطي (0.259775 ثانية، 1.325 ثانية، 0.0734 سم) على التوالي.

الكلمات الدالة:

أنظمة التعليق النشط، الأنظمة الخطية، الأنظمة غير الخطية، ربع السيارة، مسيطر التكاملي التفاضلي التناسبي.