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Effect of Taper Forms on the Dynamic Response of A Rectangular Cross Section Cantilever Beam

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ABSTRACT

Tapered beams are more effective than uniform beams because they offer a superior distribution of mass and strength and also satisfy unique functional requirements in many engineering applications. This study calculates the mode shapes and natural frequencies of straight and various tapered Euler-Bernoulli beams by finite elements using ANSYS 16.5 software. The dynamic response for a cantilever beam was obtained for different taper angles. The results were compared with the dynamic response of a straight cantilever beam, to show the effect of the taper ratio on the dynamic response of the cantilever beam, when its volume and mass are constant. The results showed that the natural frequencies were increased as the taper angle increases, and the torsional natural frequencies were shifted from the fourth natural frequency to the fifth one as the taper angle increases.

Keywords:

Tapercantilever; Natural frequency; Mode shape; Vibration; ANSYS.

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1. Introduction:

Beams are a widely popular kind of structural element. They can be categorized as uniform or tapered, slim or thick, according on their geometric characteristics. The common occurrence of uniform slender beams in literature is due to their straight forward geometry. The proper distribution of weight and strength requires tapered type, which is frequently driven by unique physical and serviceable needs. Beams are a useful model for many engineering structures and have real-world uses in constructions, helicopter blades, airplane propellers, high-speed flexible systems, and robot manipulators. The tremendous improvements in structural steel qualities over the last few decades have made structural steel the material of choice for building bridges, stadiums, factories, and tall structures [1]. Because of their structural effectiveness, which can result in significant material savings, and their ability to meet architectural and functional criteria, as well as their competitive fabrication prices, tapered members are frequently utilized in the modern construction industry. However, a designer can fully benefit from the advantages of beam tapering only if he is armed with trustworthy and effective analytical techniques that yield precise predictions of the structural behavior of the tapered component. The significance of this paper is to show the effect of different tapered angle on the dynamic response of the cantilever beam without any increase or decrease in materials. The novelty is to compare many cases of cantilever beam with different taper ratio and to keep the volume, mass and density constant for all cases.

Determining the dynamic modal features of beams, including their natural frequencies and corresponding mode shapes, is crucial for both performance evaluation and design, and it has been the focus of numerous studies. A new form of tapered beam with exponentially increasing thickness that is sitting on a linear foundation is examined for free vibration by Boreyri et al [2]. The differential transform method, (DTM) a semi-analytical method, is the foundation for the solution. The response is then obtained by solving the algebraic equations that were created by applying DTM to the nonlinear partial differential equations of the beam with variable thickness. The coupled displacement field (CDF) method is used to determine the linear and non-linear fundamental frequency parameter values of the tapered Timoshenko beams, and closed form expressions are derived for the frequency ratio as a function of the slenderness ratio, taper ratio, and maximum amplitude ratio for hinged-hinged and clamped-clamped beam boundary conditions [3]. Moreover, Bazoune [4] formulated the mass, elastic, and centrifugal stiffness matrices explicitly in terms of the taper ratios, to study the problem of free vibration of a rotating tapered beam. In this analysis, the effects of hub radius, two-plane tapering, and rotational stiffening are all taken into account. The motion equations are obtained, the generalized eigenvalue problem that goes along with it is defined with a suitable Lagrangian form, and it is solved for a variety of parameter variations. With all parameter modifications present, the impact of tapering on the beam's inherent frequencies is evaluated. The comparison of the results with those found in the literature reveals very good agreement. In a separate study, Attarnejad et al [5] used the Euler-Bernoulli theory of beam vibrations and Fortran90 programming, to solve the issue of free vibration of a tapered beam with elastic end rotational constraints. The natural frequencies and mode shapes for an Euler-Bernoulli beam with elastically supported ends were determined. In a related study, Al-Ansari [6] investigated the frequency of the tapered beam utilizing three different mathematical techniques. These techniques used ANSYS Workbench and the Classical Rayleigh Method (CRM), Modified Rayleigh Method (MRM), and (FEM) ANSYS (17.2). The fundamental distinction between the Classical Rayleigh Method (CRM) and the Modified Rayleigh Method (MRM) was the transformation of the tapered beam into an N-step stepped beam. To determine the natural frequencies, Lee et al. [7] employed a combination of the Runge Kutta approach and the determinant search method to numerically solve the ordinary differential equation governing the tapered beam. In addition, the transverse frequency of different cross section beams was studied by Firouz-Abadi R. D. et al. [8] by using the Wentzel, Kramers, Brillouin (WKB) approximation method to solve the governing equation of motion of the Euler-Bernoulli beam while considering the impact of axial force distribution. On the other hand, Bayat et al [9] determined the natural frequency and corresponding displacement of tapered beamsand described two innovative applications of the traditional Chinese method: the Max-Min Approach (MMA) and Homotopy Perturbation Approach (HPM),. It is explained how the vibration amplitude affects the non-linear frequency. Finally, the acquired results are compared with the precise data and presented through graphs and tables, demonstrating the efficiency and practicality of the MMA and HPM. These methods are exceedingly efficient and straightforward, and they produce solutions with a high degree of accuracy after just one iteration. As this study indicates, those techniques are expected to find widespread use in engineering challenges.

Maganti et al [10] investigated the free vibration properties of a symmetrically cross-sectioned functionally graded double-tapered rotating cantilever beam. They proposed a method to calculate the flapwise bending vibrations of rotating functionally graded double tapered beams linked to rigid hubs, considering deformation factors. Young's modulus along the thickness direction of the tapered beam varies continuously according to a power law. Lagrange's method is used to obtain the equations of motion utilizing hybrid deformation variables. The frequencies of the beam are evaluated using the Rayleigh-Ritz method. Several parameters are investigated to understand their impact on the flapwise bending natural frequencies of tapered beams. These parametersinclude the gradient of the material composition, taper ratios, hub radius ratios, and rotational speed. In their research, Raju *et al* [11] used continuum and finite element methods to evaluate large amplitude free vibrations of tapered beams. Shukla [12] employed the double tapered cantilever Euler beam's equation of motion to determine the natural frequencies of the structure. The formulation of finite elements involved the utilization of weighted residual and the Galerkin technique. For various taper ratios, natural frequencies and mode shapes are determined. The strength and dynamic response of reinforced concrete tapered beams strengthened by the near surface mounting (NSM)-CFRP technology were experimentally tested by Alali et al [13]. They study the effect of angle of inclination on the behavior of beams. In separate study, Haskul and Kisa [14] conducted a free vibration analysis of a double tapered beam with linearly increasing thickness and breadth. They obtained the stiffness and mass matrices of the beam to ascertain its inherent frequency and mode shape of the double tapered cracked beam. Using bending moment diagrams, Hantati et al [15], in another investigation, estimated the nonlinear frequencies and modes of the tapered beams as well as the corresponding stress distributions.

2. Methodology:

Consider a varying thickness cantilever beam of length L, as shown in Fig. 1. The width at any part of length may be obtained by manipulating the following equation:

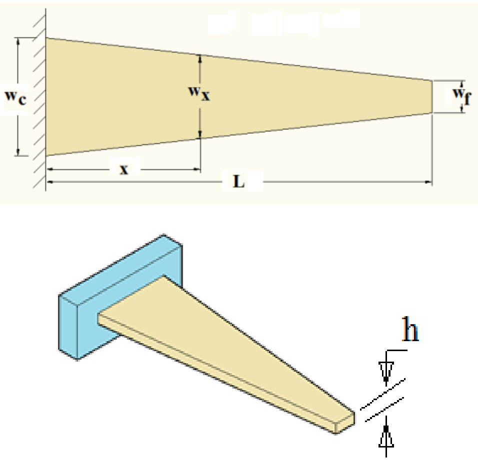


Fig. 1 Tapered cantilever beam

$$\frac{(w_c-w_f)/2}{L} = \frac{(w_x-w_f)/2}{L-x} \quad (1)$$

where w_c is the beam width at the fixed end and w_f is the beam width at the free end, while w_x is the width of the beam at a distance x from the fixed end. Rearranging this equation leads to:-

$$w_{(x)} = w_c - \frac{(w_c-w_f)}{L}x \quad (2)$$

Assume $\beta = \frac{(w_c-w_f)}{L}$ (3)

Equation (2) becomes:

$$w_{(x)} = w_c - \beta x \quad (4)$$

The cross-sectional area at a distance x from beam fixed end can be written as follow:

$$A(x) = h * w(x) \quad (5)$$

Substitute eq. (4) into eq. (5) yields:

$$A(x) = h *(w_c - \beta x) \quad (6)$$

To determine the natural frequencies and their corresponding mode shapes for the longitudinal motion of a cantilever beam with a tapered thickness, the assumption is that the cross-sectional area, which is originally plane and perpendicular to the beam's axis, remains plane and perpendicular to the axis, and that the normal stress in the axial direction is the only the component of stress. The X-axis and the beam's axis are parallel, and each section x's displacement is indicated by the symbol u. Nawal [16].

These cases can be solved using Rayleigh method to determine the natural frequency of a tapering beam. Utilizing this approach reduces the complexity associated withthe governing equation and its solution [17-19]. The general formula for the Rayleigh method is as follows:

$$\omega^2 = \frac{\int_0^l EI(\frac{d^2y(x)}{dx^2})^2dx}{\int_0^l \rho A(y(x))^2dx} = \frac{g\sum_{i=1}^{n+1} m_i\, y_i}{\sum_{i=1}^{n+1} m_i\,(y_i)^2} \quad (7)$$

In this formula, (ω) represents the frequency, (I) denotes the second moment of inertia, (E) stands for the modulus of elasticity, (ρ) represents the density, (m) denotes the mass, (A) signifies the cross-sectional area, and (y) refers to the deflection.

Due to changes in the cross-sectional area along the beam, and referring to equation (1), the second moment of inertia is the main issue with this method. Thus, the primary concept behind this effort was to transform a tapered beam into a stepped beam with (N) steps. As shown in Fig. 2, the tapered beam was separated into (N) portions of varying width.

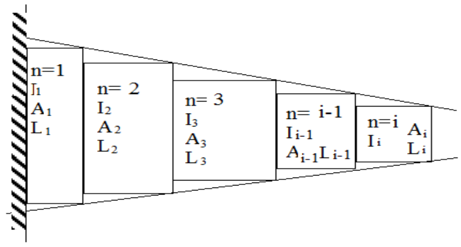


Fig. 2 Stepped Tapered Beam

The method outlined in [17] and [18] can then be used to compute the equivalent second moment of inertia.

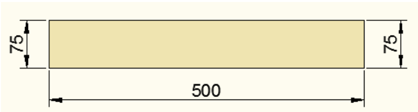
$$I_{eq} = \frac{(L_{Total})^3}{\sum_{n=1}^{N}[\frac{(L_n)^3-(L_{n-1})^3}{I_n}]} \quad (8)$$

2.1 Finite element method (FEM)

Utilizing ANSYS 16.1, the finite elements method was used in this study. The model was constructed as a one-piece cantilever beam with different taper angle or taper ratio (w_c/w_f) for each case, as shown in Table 1. The software employs a set of equation matrices equivalent to equation 7 to calculate the natural frequency for each case, considering all the given parameters. An appropriate mesh value is assumed to solve the model. The taper ratio ranged from 1 to 14, with the condition of maintain a constan volume of the beam at 0.000375 m³ (375cm), which results in constant mass. The specification of the tapered cantilever beam is shown in Table 2.

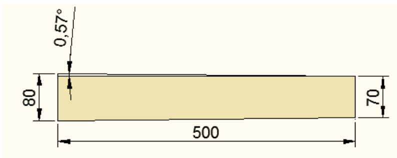
Table 1: Different cantilever beam shapes of the cases in the study

<u>Case 1 Taper 75:75</u> Taper angle = 0° Taper width ratio = 1



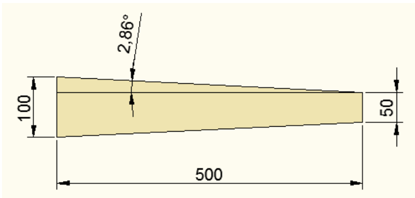
<u>Case 2 Taper 80:70</u>

Taper angle $\theta = \tan^{-1}\frac{5}{500} = 0.573°$, Taper width ratio = 80/70 = 1.143



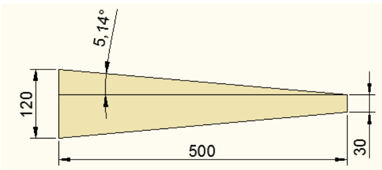
<u>Case 3 Taper 100:50 mm</u>

Taper angle $\theta = \tan^{-1}\frac{25}{500} = 2.86°$, Taper ratio 100/50 = 2



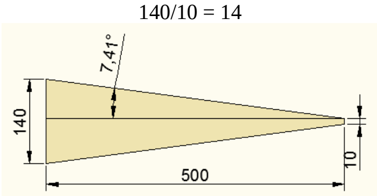
<u>Case 4 Taper 120:30 mm</u>

Taper angle $\theta = \tan^{-1}\frac{45}{500} = 5.143°$, Taper ratio = 120/30 = 4



<u>Case 5 Taper 140:10 mm</u>

Taper angle $\theta = \tan^{-1}\frac{65}{500} = 7.41°$, Taper ratio = 140/10 = 14



The beam was modeled using isotropic homogeneous structural steel with modulus of elasticity E is $200x10^9$N/m^2 and Poisson's Ratio of 0.3., Subsecquently, the model was meshed and converted to square finite elements, with a mesh size of 5 mm, ensuring accurate results. This value was recommended by the ANSYS software examples and some structural studies. A fixed support was attached to one end of the beam. The model was solved to obtain the first six mode shapes and corresponding natural frequencies. This process was repeated for various taper ratios, with model being constructed and processed in ANSYS 16.5, for each case.

Table 2: specification of the tapered cantilever beam

Case	Case1	Case 2	Case3	Case 4	Case 5
Taper angle (Degree)	0	0.573	2.86	5.143	7.41
w_c mm	75	80	100	120	140
w_f mm	75	70	50	30	10
H mm	10	10	10	10	10
L mm	500	500	500	500	500
Taper width ratio w_c/w_f	1	1.143	2	4	14
Volume cm^3	375	375	375	375	375
Density kg/m^3	7850	7850	7850	7850	7850
Mass kg	2.944	2.944	2.944	2.944	2.944
E N/m^2	$200x10^9$	$200x10^9$	$200x10^9$	$200x10^9$	$200x10^9$

3. Results and discussion

The length of the beam was (0.5) m for all cases in this paper. A total of five cases were studied, each with a different taper angle (0, 0.573, 2.86, 5.143, 7.41) and the corresponding taper ratios (w_c/w_f) (1, 1, 143, 2, 4, 14). These specific values of taper angles and taper ratio were chosen carefully to maintain a constant volumeof 375 cm^3 ensuring an accurate comparison between the cases. This leads to constant mass as the cases have the same material (structural steel) and has a density equal to 7850 kg/m^3. ANSYS16.5 was employed to solve each case and determine their respective first six natural frequencies and corresponding mode shapes. The results were compared to show the effect of taper ratio on the dynamic response of a cantilever beam. The mode shapes and their corresponding natural frequencies for all the cases are shown in Fig 3 to Fig. 7.

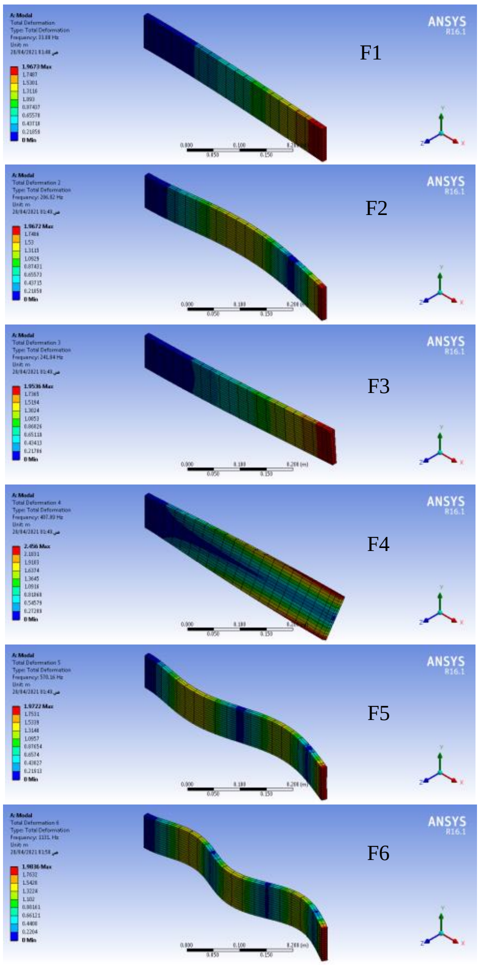
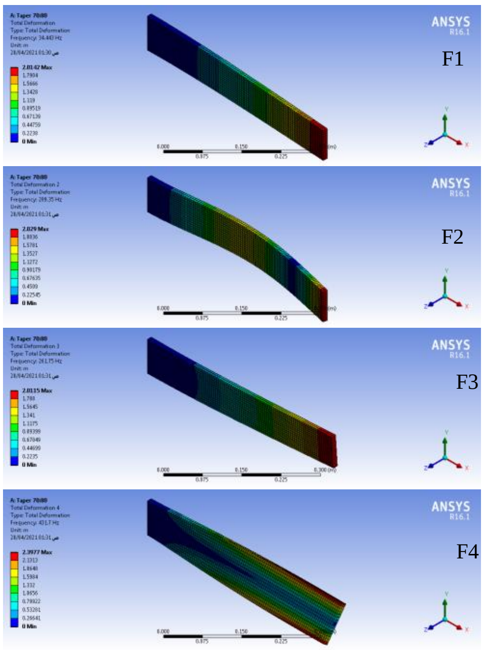


Fig. 3 First six mode shapes with their corresponding natural frequencies for 1st case with no taper cantilever beam



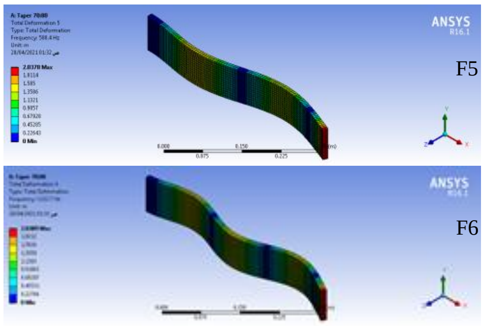


Fig. 4 First six mode shapes with their corresponding natural frequencies for 2nd case with 0.2865° taper angle cantilever beam

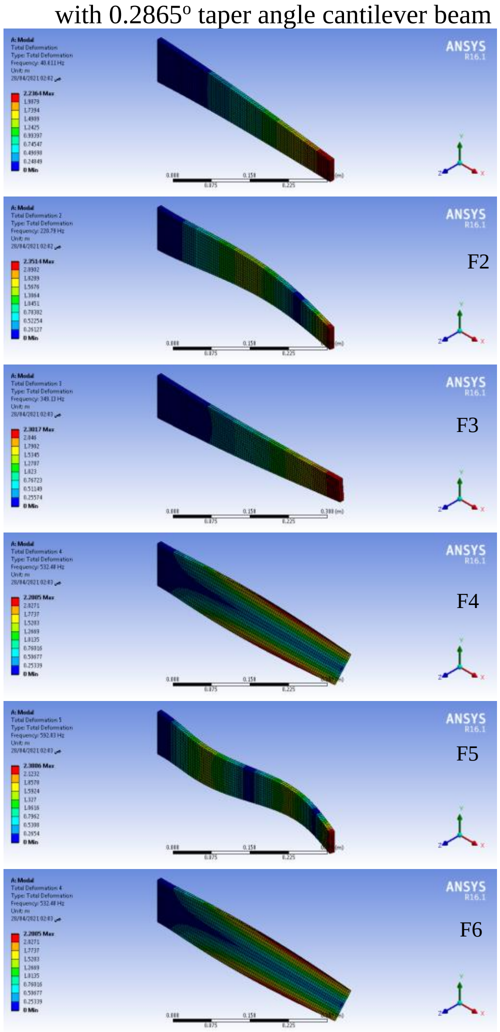
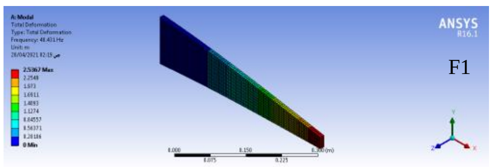


Fig. 5 First six mode shapes with their corresponding natural frequencies for 3rd case with 2.86° taper angle cantilever beam



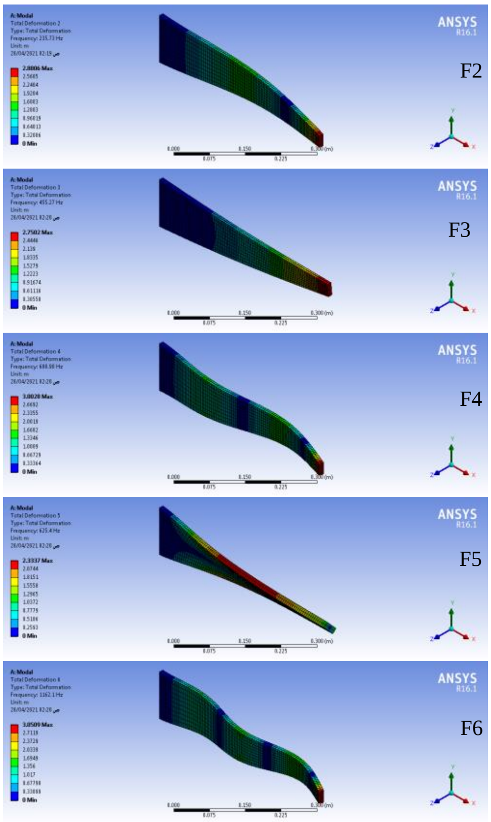
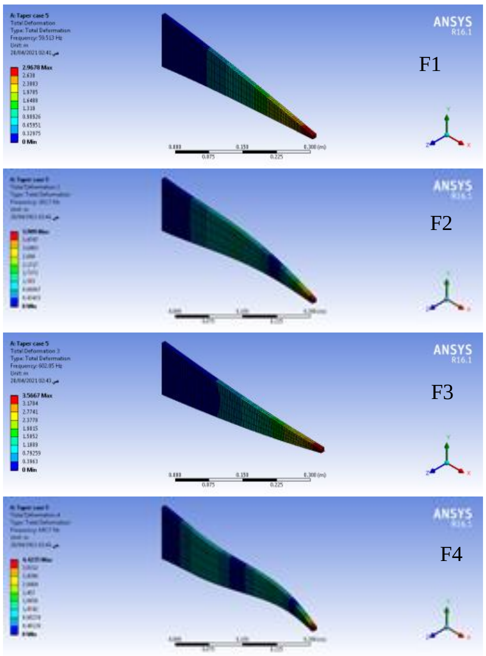


Fig. 6 First six mode shapes with their corresponding natural frequencies for 4th case with 5.143° taper angle cantilever beam



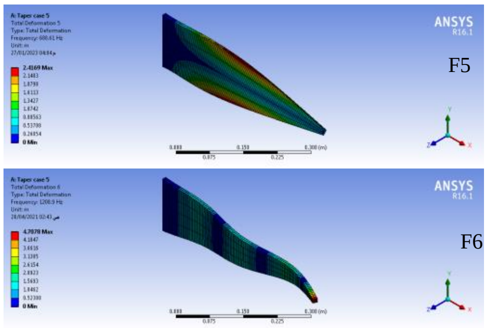


Fig. 7 First six mode shapes with their corresponding natural frequencies for 5th case with 7.41° taper angle cantilever beam

It can be observed from these figures that a torsional natural frequency has occurred among these six mode shapes. This torsional natural frequency can be seen in the fourth natural frequency for taper width ratios of1, 1.143, and 2. However, it shifts to become the fifth natural frequency when the taper width ratio has been increased to 4, and remains the fifth natural frequency for a taper width ratio of 14. This indicates a relationship between the taper width ratio and the torsional natural frequency. As the taper width ratio increases, the torsional natural frequency shifts to a higher natural frequency. This may be attributed to the shifting of the center of gravity towards the fixed end, which increases the stiffness and modulus of rigidity of the beam. The first six natural frequencies for each tapered width angle are shown in Fig. 8 to Fig. 12. The first natural frequencycalled also known as the fundamental natural frequency, increases with the taper ratio. It starts at approximatly 30 Hz for a taper ratio of 1 (no taper) and reaches 60 Hz for a taper ratio of 14. The second natural frequency is significantly higher than the first, exceeding 200 Hz for each case.

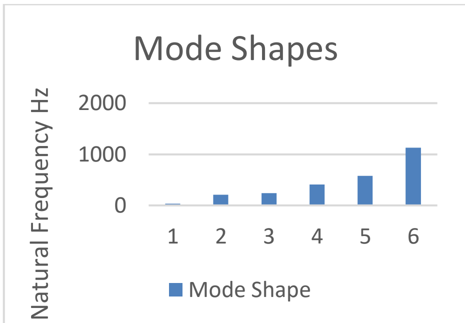


Fig. 8 Mode Shapes and natural frequencies for cantilever taper beam angle 0°

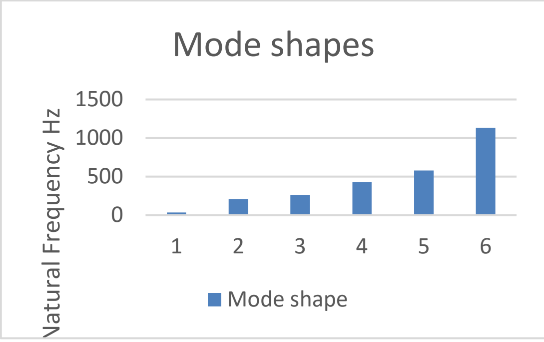


Fig. 9 Mode Shapes and natural frequencies for cantilever taper beam angle 0.573°

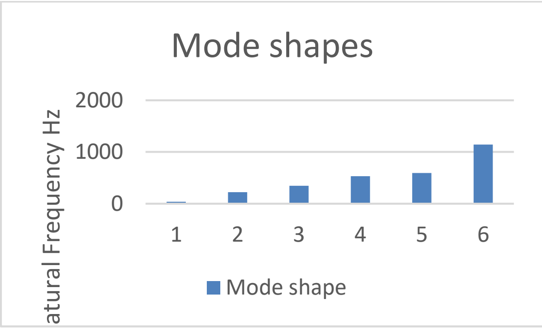


Fig. 10 Mode Shapes and natural frequencies for cantilever taper beam angle 2.86°

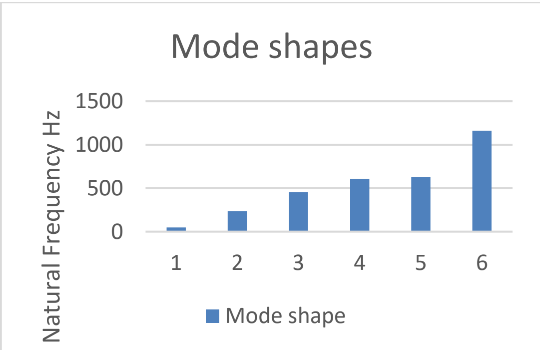


Fig. 11 Mode Shapes and natural frequencies for cantilever taper beam angle 5.143°

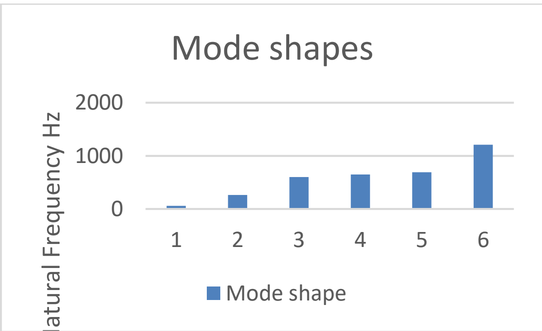


Fig. 12 Mode Shapes and natural frequencies for cantilever taper beam angle 7.41°

From Table 3 and Fig. 13, it can be shown that the six natural frequencies increased as the taper angle increased. The third and fourth natural frequencies increased rapidly due to the increase in taper angle, and the fundamental natural frequency approximately doubled between the 0° and 7.41°. The higher natural frequencies, F5 and F6, were less affected by the change in taper angle. It may be attributed to the higher frequencies of the tapered cantilever beam compared to the uniform beam for some reasons. First, the tapered beam is stiffer at the fixed end as this end is bigger than the free end. Second, because the tapered beam has less mass near its free end than the uniform beam, the inertial loading on it is smaller.

Table 3: Natural frequency for various taper angle

Taper angle (Degree)	Taper width ratio w_1/w_2	F1 (Hz)	F2 (Hz)	F3 (Hz)	F4 (Hz)	F5 (Hz)	F6 (Hz)
0	1	33.08	206.82	241.84	407.89	578.16	1131
0.573	1.143	34.443	209.35	261.75	431.7	580.4	1132.7
2.86	2	40.611	220.79	349.13	532.48	592.03	1144.3
5.143	4	48.431	235.73	455.27	608.98	625.4	1162.1
7.41	14	59.513	262.7	602.05	647.7	688.61	1208.9

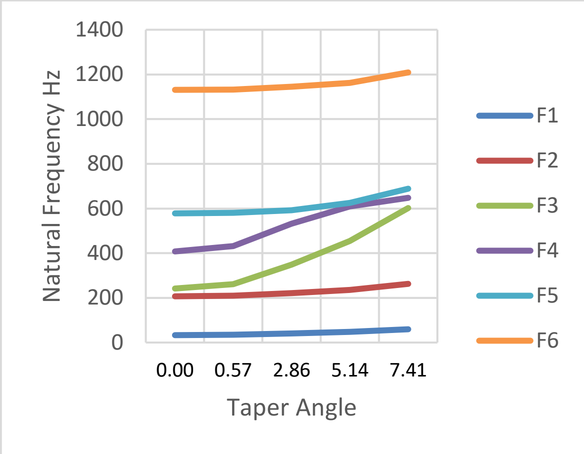


Fig. 13 Effect of taper angle on the natural frequency

4. Conclusion

The finite element method was used to investigate the effect of different tapered width ratio on the natural frequency and mode shape of a cantilever beam. The results indicate that taper ratio has an impact on both the natural frequency and the mode shape. Specifically, as the taper ratio increases, the natural frequency also increases with the increase of the taper ratio. This can be attributed to the shifting of the center of gravity towards the fixed point, which increases the stiffness of the beam. It is important to note that this study ensure that the taper ratio does not alter the volume or the mass of the beam, ensuringaccurate results.

Moreover, the torsional natural frequencies exhibited a shifted from the fourth natural frequency to the fifth as the taper ratio increased from 2 to 4. it can be observed that the torsional natural frequency continues to shift to higher value with further increases in the taper ratio. This behavior can be attributed to the same reasons mentioned earlier, along with an increase in rigidity, which affects the torsional natural frequency.

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تأثير الأشكال المستدقة على الاستجابة الديناميكية لعتبة ناتئة ذات مقطع عرضي مستطيل

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الملخص

تعتبر العتبات المستدقة أكثر فاعلية من العتبات المستقيمة لأنها توفر توزيعًا فائقًا للكتلة والمتانة ، كما أنها تلبي المتطلبات الوظيفية الفريدة في العديد من التطبيقات الهندسية. تحسب هذه الدراسة أشكال الوضع والترددات الطبيعية لعتبات أويلر- برنولي المستقيمة والمستدقة بواسطة طريقة العناصر المحدودة باستخدام برنامج ANSYS 16.5. تم الحصول على الاستجابة الديناميكية لعتبة ناتئ لزوايا مستدقة مختلفة. تمت مقارنة النتائج مع الاستجابة الديناميكية لعتبة ناتئ مستقيمة ، لإظهار تأثير نسبة الاستدقاق على الاستجابة الديناميكية لعتبة ناتئ ، عندما يكون حجمها وكتلتها ثابتًا. اظهرت النتائج زيادة الترددات الطبيعية قد زادت مع زيادة زاوية الاستدقاق ، وحصل زحف للترددات الالتوائية الطبيعية من التردد الرابع إلى الخامس مع زيادة زاوية الاستدقاق.

الكلمات الدالة :

عتبات ناتئة مستدقة؛ تردد طبيعي؛ شكل الوضع؛ اهتزاز؛ أنسز.