The Effect Of Center Distance Change On Gear Teeth Engagement And Stress Analysis

Omar D.Mohammad

Assistant Lecturer- College of engineering \ University of Mosul
Enggomar@yahoo.com

Abstract

The present study concentrates on the changes in gear teeth engagement and stress analysis of meshing teeth when gearing system is operated at a non-standard extended center distance. (Ansys) programming using F.E.M have been applied for stress analysis on a gear model. Many cases with changing center distance have been studied. It is clear that the operating center distance was increased; the stresses generated on tooth will be increased dependently.

Keywords : Spur gear, Center distance, Teeth engagement, Stress analysis

تأثير تغير مسافة المراكز على تعشيق أسنان التروس وتحليل الإجهادات المتولدة فيها

عمر داود محمد

مدرس مساعد - كلية الهندسة / جامعة الموصل
Enggomar@yahoo.com

الخلاصة

يتمحور هذا البحث حول التغييرات التي تطرأ على عملية تعشيق الأسنان وتحليل الإجهادات المتولـدة فيهـا عنـد تشـغيل نظـام التـروس بمسـافة مراكـز غيـر قياسـية. وقـد تـم إسـتخدام برمجة (Ansys) التي تعتمد طريقة العناصر المحددة في تحليل الإجهادات لنموذج الترس. وقد تمت دراسة عدد من الحالات عند تغير مسافة المراكز. ويتضح من هذه الدراسة أنه بزيادة مسافة المراكز التشغيلية تزداد قيم الإجهادات المتولدة على السن المعشق.

Received 30 May 2007 **Accepted 2 Jan. 2008**

Symbols

ARC' : Operating arc of contact (mm)	d'_p : Operating pinion pitch circle (mm)
B: Increase in backlash (mm)	G: gear ratio.
C: Standard center distance (mm)	p'_c : Operating circular pitch (mm)
C' : Operating center distance (mm)	PTH' : Operating path of contact (mm)
$C.R'$: Operating contact ratio.	ΔC : Change in center distance (mm)
d'_g : Operating gear pitch circle (mm)	ϕ' : Operating pressure angle (degree)

Introduction

Gears may be designed to operate at a non-standard center distance to introduce backlash, to accommodate space constraints and to adjust for anticipated deflections under load as well as geometry changes due to thermal effects [1]. Then for proper operation, center distance must be maintained within predetermined tolerances. Since the center distance is a machined dimension, it may not come out to be exactly what the design calls for. Moreover, due to mounting inaccuracies, misalignment and other faults created during running of gearing system, the desired center distance is practically not possible to accurately achieve or maintain. So center distance may increase over its standard value, it is important that the increase of center distance may causes many problems as will be explained later.

Lin, Lion, Oswald and Townsend [1] presented an analytical study on using hob offset to balance the dynamic tooth strength of spur gears operated at extended center distance. In this study stress analysis had been done on modified involute gear teeth due to hob offset.

This research work concentrates on studying the changes in gear teeth engagement and stress analysis of standard involute meshing teeth when gearing system is operated at a non-standard extended center distance. In this study many cases of center distance changing had been depended and studied. In each case stress analysis had been found.

Gear model

This study using F.E.M had been applied on a pinion gear model running at (5000 r.p.m) transmitting (400 kW). All data relating to pinion and gear is shown in table (1).

Gear Teeth Engagement

To satisfy the fundamental law of gearing, that is, to maintain a constant velocity ratio of a pair of intermeshing gears, the tooth curves are to be designed that the common normal to the tooth profiles at the point of contact will always pass through the pitch point. If the gear tooth form is *not* an involute, then an error in center distance will violate the fundamental law, and there will be variation or ripple in the output velocity. The output angular velocity will not be constant for a constant input velocity. However, with an involute gear tooth form, the fundamental law of gearing still holds in modified center distance case, and then the center distance can be changed without affecting the angular velocity ratio. This property is the principal advantage of the involute over all other possible tooth forms and the reason why it is nearly universally used for gear teeth [2] and [3]. For operating at a non-standard center distance, operating gives a set of design parameters will be assigned by the term "*operating*".

The pitch circle only comes into being when we mate this gear with another to create a pair of gears or gear set, then the operating pitch circle can be introduced that differs from the

Table (1): Data relating to pinion and gear

		Pinion	Gear
(Teeth no.) Z		50 teeth	100 teeth
(Running speed) N		5000 r.p.m	2500 r.p.m
(Pitch circle diameter) P.C.D		200 mm	400 mm
(Gear ratio) G		2.0	
Alloy designation	**DIN**	1.7218 25CrMo4	
	AISI	4130	
(Yield stress) σ_y		570 MN/m^2	
(Modulus of elasticity) E		207 GN/m^2	
(Poisson's ratio) υ		0.3	
(Module) m		4 mm	
(Pressure angle) ψ		20°	
(Addendum) $h_a = m$		4 mm	

(Dedendum) $h_d = 1.25\text{m}$	5 mm
(Root fillet) r_f	1 mm
(Face width) B	40 mm

nominal pitch circle that given by mounting the gear set at the standard center distance. For operating at the *operating* center distance, pitch circles will be increased in so way maintaining a constant velocity ratio [2], see figure (1). The operating pitch circles for gear and pinion can be defined as following [4]:

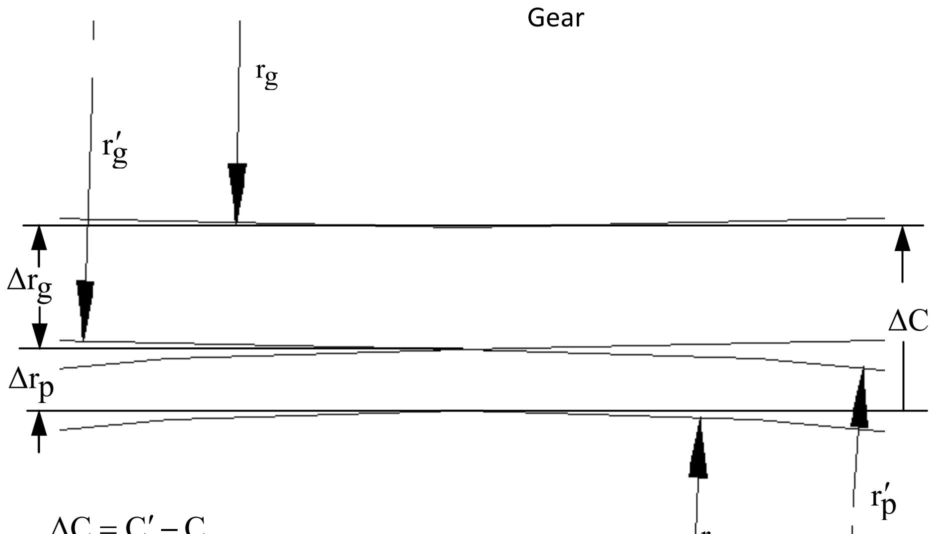
$$d'_p = \frac{2C'}{G+1} \qquad (1)$$

$$d'_g = \frac{2C'G}{G+1} \qquad (2)$$

$$\frac{d_g}{d_p} = \frac{d'_g}{d'_p} = G \qquad (3)$$

For involute gearing, as the center distance increase, the base circles remain unchanged, also the common normal (pressure line) is still tangent to the two base circles and still goes through the pitch point, see figures (2) and (3).

The pressure angle can be defined as the angle measured between the pressure line and the common tangent to the pitch circles. It is convenient to consider the pressure angle at the pitch point (point of intersection of the line of action and the line of centers).



The pressure angle can be related with the base circles and the pitch circles as following:

$$r_b = r.\cos(\phi)$$

Then, $\cos(\phi) = \frac{r_b}{r}$ ....(4)

As the center distance increases, so will the pressure angle and vise versa [4], see figures (2) and (3).

$$\cos(\phi') = \frac{r_b}{r'} \text{.......................................(5)}$$

From (4) and (5),

$$\cos(\phi') = \frac{\cos(\phi).r}{r'} \qquad (6)$$

Also, equation (6) can be written in term of center distance as following [4]:

$$\cos(\phi') = \frac{\cos(\phi).C}{C'} \qquad (7)$$

Radii in above equations may be for gear or pinion.

Table (2), shows the operating pitch circles and operating pressure angle for all studied cases.

Table (2): Pitch circles and pressure angle for all studied cases.

Case no.	ΔC (mm)	C' (mm)	d'_p (mm)	d'_g (mm)	ϕ' (degree)
1	0.0	300.0	200.00	400.00	20.00
2	0.5	300.5	200.33	400.66	20.26
3	1.0	301.0	200.66	401.33	20.51
4	1.5	301.5	201.00	402.00	20.77
5	2.0	302.0	201.33	402.66	21.01
6	2.5	302.5	201.66	403.33	21.26
7	3.0	303.0	202.00	404	21.50

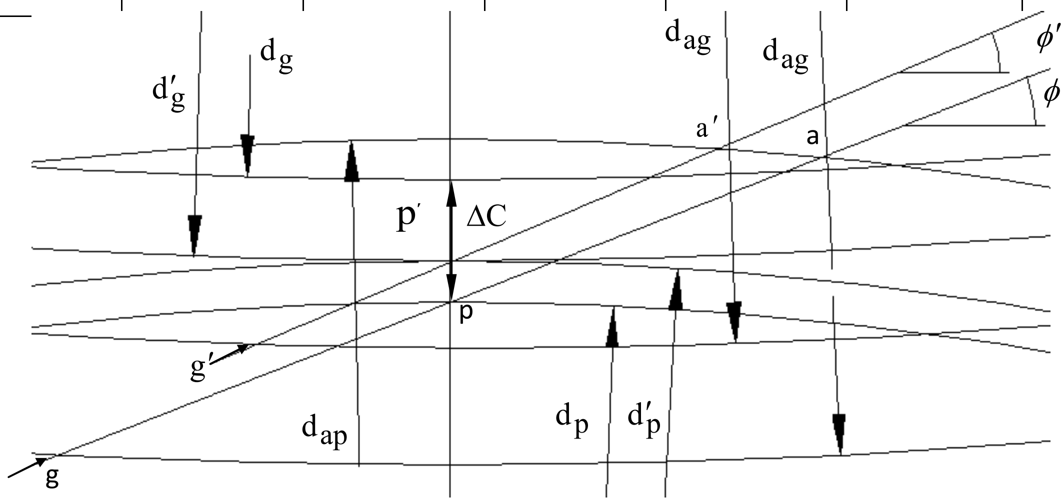
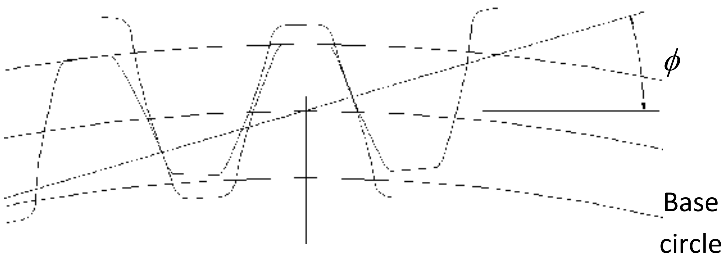
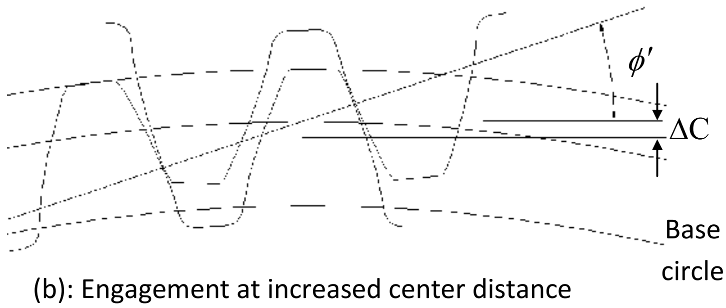


Figure (2): change of circles and pressure angle with changing center distance.



(a): Engagement at standard center distance



(b): Engagement at increased center distance

Figure (3): Engagement change with increasing center distance

The path of contact can be defined as following [5]

$$PTH = \sqrt{r_{ap}^2 - r_p^2 \cos^2(\phi)} + \sqrt{r_{ag}^2 - r_g^2 \cos^2(\phi)} - (r_p + r_g)\sin(\phi) \quad \text{..........}(8)$$

From figure (2), it can be observed that the addendum circles will remain unchanged, as these circles are machined dimensions. Hence the path of contact is then decrease. The *operating* path of contact can be defined as following:

$$PTH' = \sqrt{r_{ap}^2 - r_p'^2 \cos^2(\phi')} + \sqrt{r_{ag}^2 - r_g'^2 \cos^2(\phi')} - (r_p' + r_g')\sin(\phi') \quad \text{.....}(9)$$

Then, the arc of contact can be defined as following:

$$ARC' = \frac{PTH'}{\cos(\phi')} \quad \text{........................}(10)$$

As the pitch circle increase, so will the circular pitch and vise versa.

$$p_c' = \frac{\pi . d'}{z} \quad \text{...................................}(11)$$

Operating at a non standard center distance changes the nature of teeth engagement, the gear teeth meshing will then be decreased. Hence the contact ratio will be decreased too.

$$CR' = \frac{ARC'}{p_c'} \quad \text{................................}(12)$$

Then for all studied cases, the contact ratio can be calculated. See table (3).

Table (3): Operating contact ratio for all studied cases.

Case no.	PTH′ (mm)	ARC′ (mm)	p'_c (mm)	CR′
1	21.28	22.64	12.56	1.80
2	19.83	21.14	12.58	1.68
3	18.40	19.65	12.60	1.56
4	16.92	18.10	12.62	1.44
5	15.58	16.69	12.65	1.32
6	14.19	15.22	12.67	1.20
7	12.83	13.78	12.69	1.08

It is important to note that the contact ratio in the last case approached from unity (the minimum possible contact ratio), then any increase in center distance will result in unpractical engagement.

Backlash

Backlash can be generally defined as the clearance between mating teeth in assembled condition. It is the amount by which the width of a tooth space exceeds the thickness of the meshing tooth along the circumference of the pitch circle.

The main purpose of providing backlash is to prevent jamming and to ensure that no contact is made on both sides of the teeth simultaneously. Proper amount of backlash ensures smooth running of the gearset. Too little backlash may lead to overloading, overheating, jamming and ultimately seizure and eventual failure of the system. Moreover, a tight mesh may result in objectionable noise during running. On the other hand, excessive backlash may cause non-uniform transmission of motion especially if the amount of backlash varies from tooth to tooth due to machining and other errors. Excessive backlash may also cause noise and impact loads in case of reversible drives.

Backlash might occur in two ways: (1) if the tooth thickness is less than one-half of the circular pitch and (2) if the operating center distance is greater than its standard value. Increasing the center distance will increase the backlash and vise versa [6].

For operating at a non-standard center distance, increase in backlash (B) can be found as in equation (13). Table (4) shows values of (B) for all studied cases.

From AGMA standards [4], allowable backlash for the studied gearset is limited to the value of (0.4 mm). It is clear that the backlash values exceed its allowable value at case (3), and then excessive backlash may results in problems in load transmission.

$$B = 2C'(\mathrm{inv}\,\phi' - \mathrm{inv}\,\phi) \qquad (13)$$

where :

$\mathrm{inv}\,\phi' = \tan\phi' - \phi'$

$\mathrm{inv}\,\phi = \tan\phi - \phi$

ϕ' and ϕ are in radians.

Table (4): increase in backlash for studied cases

$\phi = 20^\circ$ **(standard pressure angle)**			
Case no.	C' (mm)	ϕ' **(degree)**	**B (mm)**
1	300.0	20.00	0.00
2	300.5	20.26	0.36
3	301.0	20.51	0.72
4	301.5	20.77	1.08
5	302.0	21.01	1.45
6	302.5	21.26	1.89
7	303.0	21.50	2.28

Interference

Simple decrease in the center distance may result in interference problem. Then operating at a non-standard center distance (increasing the center distance) reduce the interference problem as the addendum circle of gear shifts away from base circle of pinion. On the other hand, increasing the center distance results in decreasing in the contact ratio as explained earlier, and then gear carrying capacity will decrease.

Load Sharing

From figure (4) it is clear that the first point of action “g” which is located on the first meshing tooth is associated with point “c” which is located on the second meshing tooth of the same pinion. Therefore the load will be shared between these two points, similarly points “f” and “b”, also “e” and “a”. Then point “a” will goes out of contact, therefore the full load will be applied on point “e”, also point “d” until the contact being at point “c”, then a new meshing tooth comes in to contact. In low contact ratio gearing, and when a single pair of teeth is engaged, this pair transmits the full load or the full load is then applied on the one meshing tooth only. Almost, critical conditions (for maximum generated root stresses) occur in the one pair contact zone. When double pair of teeth are engaged, the transmitted load will be divided between two meshing teeth. Practically the load is not divided fairly; load sharing depends on contact ratio value and stiffness of meshing tooth at point of application of load [7]. In figure (5) the load sharing is drawn against the path of contact for low contact ratio sharing. Meshing at point “c” creates the largest possible bending moment with a single tooth contact; then point “c” is called the highest point of single tooth contact. It is clear that the meshing at point “c” with full applied load on meshing tooth leads to a maximum generated stresses in root area, hence the load applied on point “c” is the *critical* load. To determine location of point “c” on path of contact, (S) must be calculated, and then the angular position of meshing tooth can be measured by angle(γ), as shown in figure (6).

When the center distance increases then the path of contact and the contact ratio will be decreased, as explained earlier. Hence the location of the critical load will then be moved towards tip of the meshing tooth, as the single tooth contact zone changes with changing load sharing, see figure (7).

Gears are used to transmit mechanical power and this requires applying a mechanical torque that can be calculated as the following, [3]:

$$T = \frac{P}{\omega} \text{ ....(14)}$$

The normal load applied on meshing teeth can be found as the following, [3]:

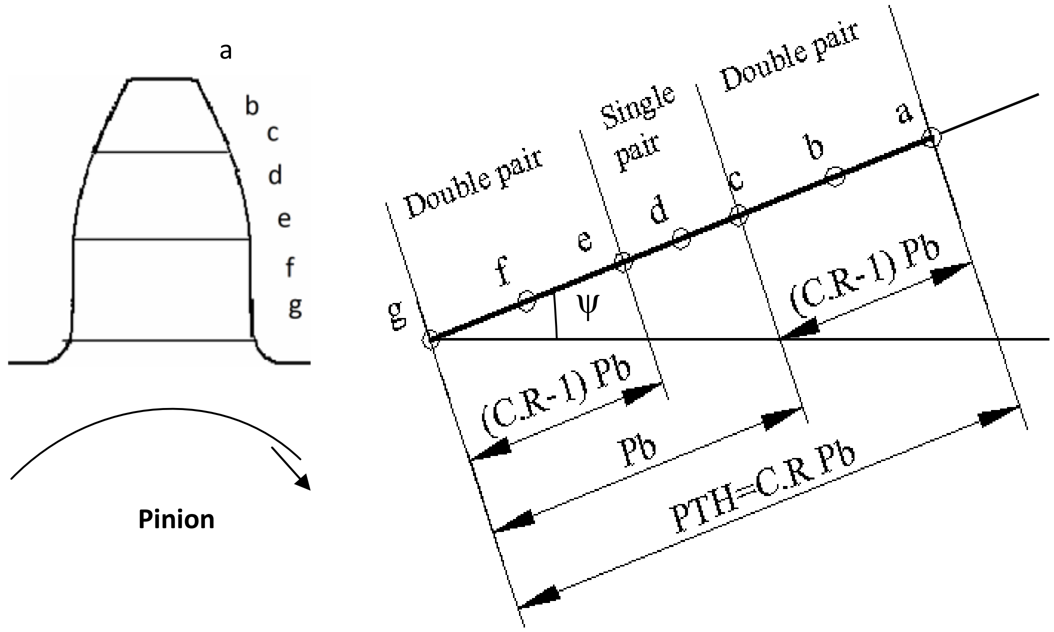


Figure (4): Contact zones for low contact ratio.

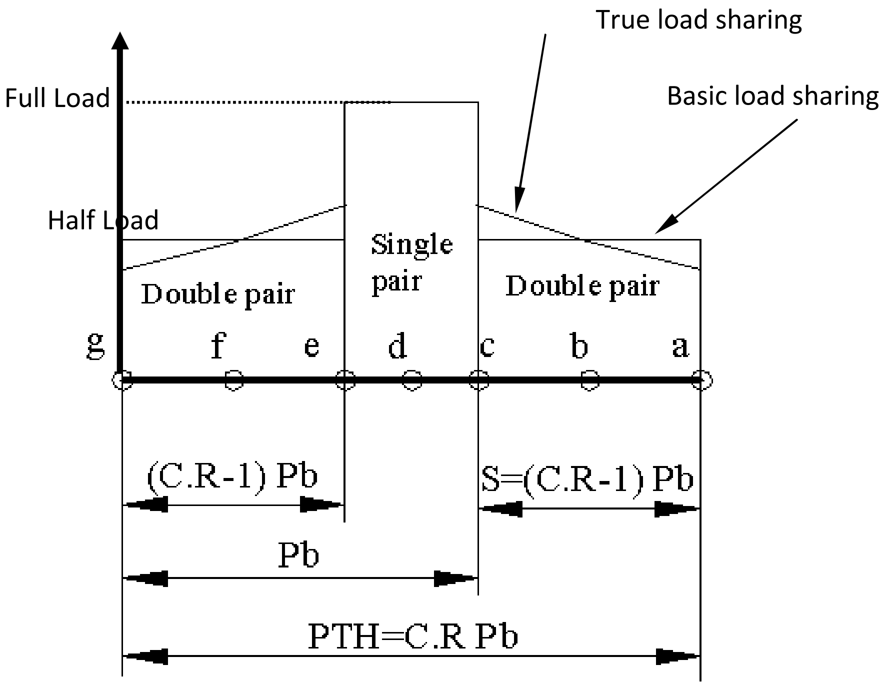


Figure (5): Load sharing for low contact ratio

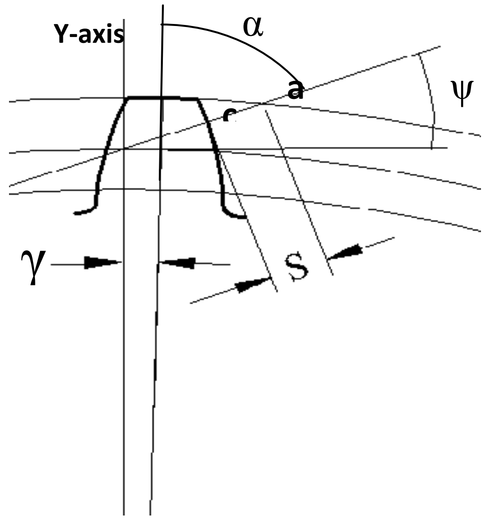


Figure (6): Position of meshing tooth for critical load.

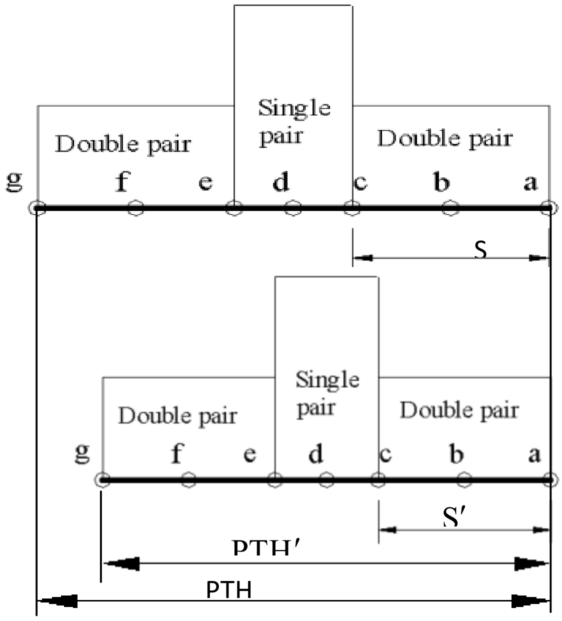


Figure (7): Changing load sharing

Transmitted load

$$F = \frac{T}{r_b} \quad ....(15)$$

The problem of stress analysis in this study is assumed as a plane elastic problem, since the applied transmitted load is assumed to be uniformly distributed across the width of the meshing tooth. Therefore the load had been depended per unit width of tooth as following:

$$F_n = \frac{F}{B} \quad ....(16)$$

As the center distance increases, the component (F_x) and (F_y) will be

$$F_x = F_n . \cos(\phi') \quad ....(17)$$

$$F_y = F_n . \sin(\phi') \quad ....(18)$$

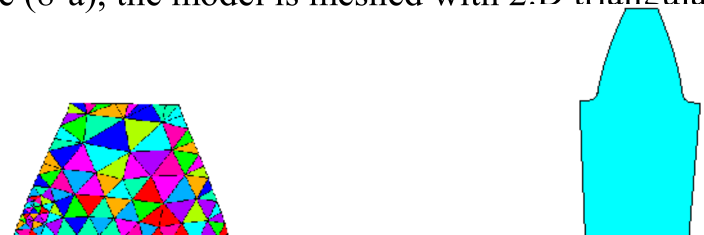
Table (5) shows all data required for determining the critical conditions of loading for all studied cases of contact ratio.

Table (5): The data required for determining the critical conditions.

Case no.	CR'	S' (mm)	(x , y) of point "c"	γ (degree)
1	1.80	9.44	-0.86 , 100.31	1.24
2	1.68	8.02	-1.74 , 100.80	0.63
3	1.56	6.60	-2.62 , 101.31	0.02
4	1.44	5.19	-3.49 , 101.82	-0.58
5	1.32	3.77	-4.37 , 102.34	-1.21
6	1.20	2.36	-5.25 , 102.87	-1.80
7	1.08	0.94	-6.13 , 103.41	-2.44

Ansys Programming

Ansys package is one of the efficient engineering programming, which has the ability to solve many engineering problems using F.E.M. This programming has easiness in use, flexibility in application and reliability for solving problems in engineering fields such as stress analysis field. The tooth model in this study had been assumed as a plane stress problem due to small width of tooth compared with radius of gear. As shown in figure (8-a), the model is meshed with 2.D triangular 6-node



elements. All nodes on the three sides (A, B, C) of the model indicated in figure (8-b) are kept fixed as boundary conditions for the solution. For each case of the center distance change, the load is applied at locations (x , y) when tooth angular position make (γ), as indicated in table (5).

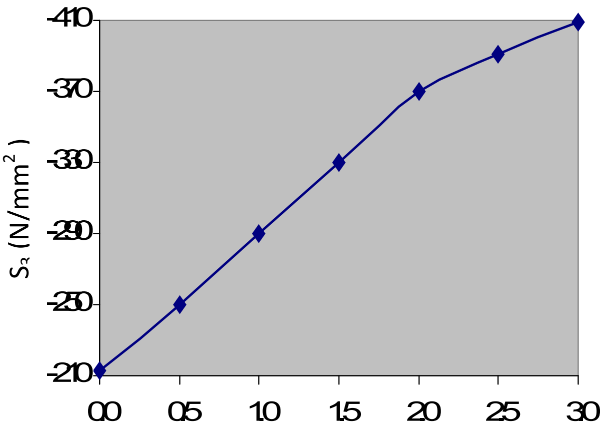
Results and Discussions

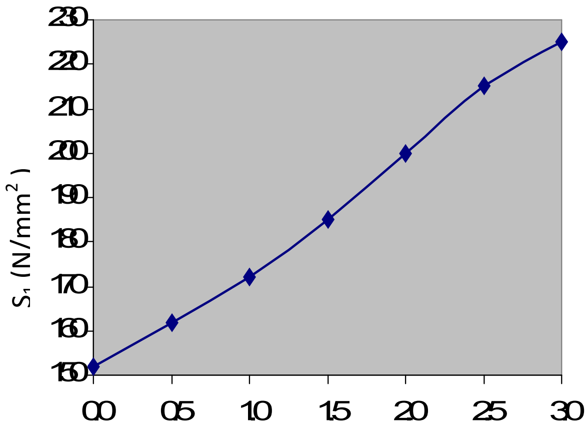
Curves plots can be used for observing the changes of stresses generated in the tooth model with a changing of center distance.

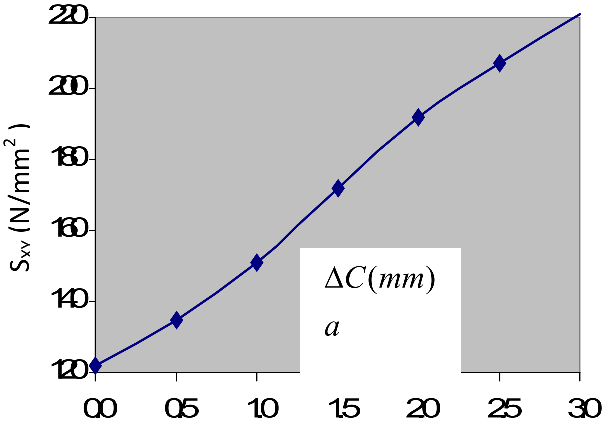
Contact area: Figures (9-a), (9-b) and (9-c) show the change of (S_3), (S_{XY}) and (S_V) respectively against the center distance change for the contact area. From these plots, it is clear that the maximum generated stresses in this area increases with increasing the center distance, this due to decrease of direction of applied critical load relative to the axis of the meshing tooth (decrease of "α" that is indicated in figure (6)).

Root Area: Figures (10-a), (10-b) and (10-c) show the changes of (S_1), (S_{XY}) and (S_V) respectively against the center distance change for the root area. From these plots, it is clear that the maximum generated stresses in this area increase with increasing the center distance, this due to increase of force moment (increase the distance measured between the point of application of critical load and the root of meshing tooth).

Each one of the three previewed stresses is considered as a criterion of one of the theories of failure. However the three stresses increased with increasing center distance, then failure is expected for increasing center distance.

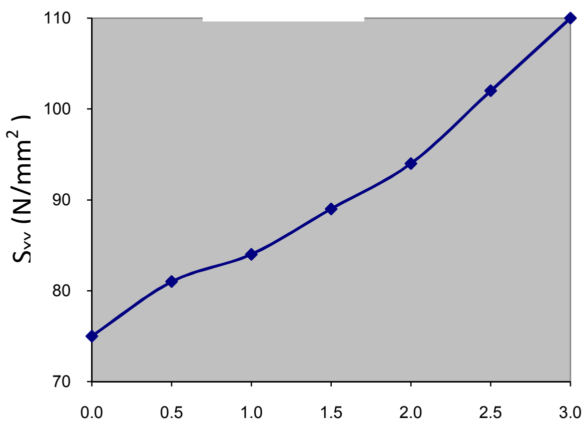






ΔC(mm)

a



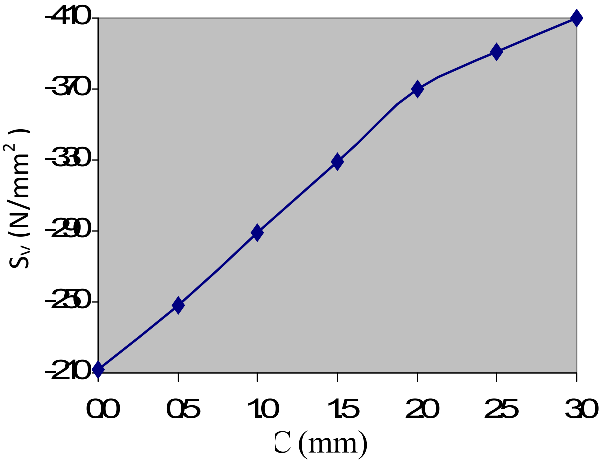


Figure (9): Change of stresses at contact area with center

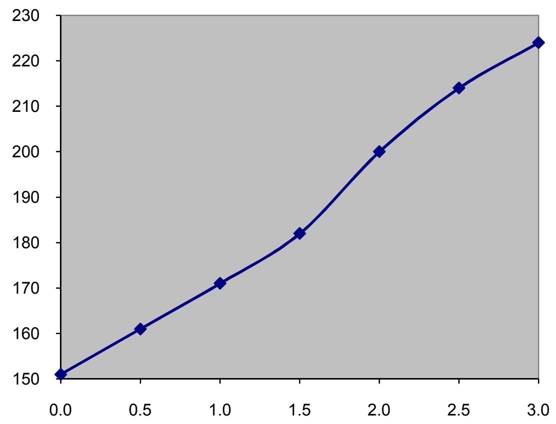
Sv (N/mm²)

ΔC(mm)

b

ΔC(mm)

b



$\Delta C(mm)$

c

$\Delta C(mm)$

c

Figure (10): Change of stresses at root area with center

Conclusions

1. The stresses generated on gear teeth change with changing operating center distance of gearing, where as the maximum generated stresses (observed in contact and root areas) increase with increasing center distance.

2. Failure of gear teeth is expected for operating at a non standard increased center distance
3. Load carrying capacity reduces with increasing the center distance as the contact ratio decreases
4. Backlash may increase excessively with increasing the center distance then problems may occur in power transmission.
5. Operating at the standard center distance results in the best power transmission condition from the point view of stress analysis, load carrying capacity, backlash, and interference.

References

1. Lin H.,Liou C., Oswald F., and Townsend D., "Balancing Dynamic Strength of Spur Gears Operated at Extended Center Distance", International Power Transmission and Gearing Conference, U.S.A, California, 1996.
2. Norton R.L., "Design of Machinery", McGraw-Hill Companies, Inc, U.S.A., NewYork, 2001.
3. Maitra G.M., "Handbook of Gear Design", Tata McGraw-Hill Company, India, NewDelhi, 1997.
4. Dudley D.W., "Gear Handbook", McGraw-Hill Company, Inc, U.S.A., NewYork, 1962.
5. Hannah J., and Stephens R.C., "Mechanics of Machines", Edward Arnold, Ltd., U.K., London, 1975.
6. Hamrock B., Jacobson B., and Schmid S., "Fundamentals of Machine Element", McGraw-Hill Companies, Inc, U.S.A., NewYork, 1999.
7. Elkholy A.H., "Tooth Load Sharing in High Contact Ratio Spur Gears", Journal of Mechanisms, Transmissions, and Automation in Design", Vol. 107, No. 1, pp. 11-16, 1985.

The work was carried out at the college of Engg. University of Mosul