

Validation Of The Total Resistance Heat Dissipation Model For Heat Transmission Through Annular Fins With Variable Heat Transfer Coefficient

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Abstract

The present paper includes analytical investigation of the validity of total resistance heat dissipation model using variable heat transfer coefficient for annular fins constant thickness. The model is given as:

$$TR_{th} = \sqrt{\left[\left(\frac{\ln(r_o/r_i)}{2\pi KW} \right)^2 + \left(\frac{W}{4\pi K(r_o^2 - r_i^2)} \right)^2 \right]} + \frac{(ro - ri)^{am} (am + 1)(am + 2)}{2\pi ka [(am + 2)(ro - ri)^{am+1} ro - (ro - ri)^{am+2}]}$$

Furthermore, a finite difference method using SOR technique is devised to serve the two purposes of verifying the two dimensional heat transfer model and to cover a wide range of fin parameters and heat transfer coefficient models. The results were in agreement and proved the validity of the suggestion model under the both the constant and variable heat transfer coefficient assumptions.

Key words: Total resistance, Variable heat transfer coefficient, Heat transfer, Annular fins

تحقيق نموذج المقاومة الكلية لانتقال الحرارة في الزعانف الحلقية باستخدام معامل انتقال حرارة متغير

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الخلاصة

تضمن البحث الحالي إجراء تحقيق لصحة نموذج المقاومة الكلية لانتقال الحرارة تحليلياً باستخدام معامل انتقال حرارة متغير لزعنفة حلقية ثابتة السمك. النموذج ممكن إعطائه كالاتي:

$$TR_{th} = \sqrt{\left[\left(\frac{\ln(r_o/r_i)}{2\pi KW} \right)^2 + \left(\frac{W}{4\pi K(r_o^2 - r_i^2)} \right)^2 \right]} + \frac{(ro - ri)^{am} (am + 1)(am + 2)}{2\pi ka [(am + 2)(ro - ri)^{am+1} ro - (ro - ri)^{am+2}]}$$

ولذلك تم استخدام تحليل الفروق المحددة باستخدام تقنية SOR كحل إضافي ليلائم الغرضين نموذج انتقال الحرارة بالاتجاهين ولتغطية مدى واسع من متغيرات الزعنفة ومعامل انتقال الحرارة. ظهرت النتائج متوافقة ومن ثم إثبات صحة النموذج الرياضي للمقاومة الكلية لانتقال الحرارة عند فرضيات معامل انتقال حرارة ثابت ومتغير.

الكلمات المفتاحية: المقاومة الكلية ، معامل انتقال حرارة متغير ، انتقال الحرارة ، الزعانف الحلقية

1. Introduction:

The past century witnessed great advances in the art of heating and cooling of air. Prior to this period, prime surfaces were employed almost exclusively. Today, prime surfaces are largely replaced by extended or finned surfaces where such extended surfaces are used to enhance the rate of heat transfer from the primary surface. The selection of any particular type of fin depends on the geometry of the prime surface it is used on.

Radial or concentric annular fins are among the most common choices for enhancing heat transfer from outer surfaces of circular tubes. In the conventional heat transfer analysis of radial fins, it is a standard practice to assume that the convection heat transfer coefficient at the fin surface is uniform. However, the hydrodynamics of any flow through the fins passages and the variation of the surface temperature of the fins dictates that the convective heat transfer coefficient, whether it's natural, forced or mixed, can be anything but uniform. This is believed to have great impact on both the total heat dissipation and the temperature distribution along the fin.

Irey [1] investigated circular fins and reported that the one - dimensional approximation is only valid for small Biot number ($Bi=hr/k$).

Keller and Somers [2] presented an analytical solution for annular fins with two dimensional heat flow, however, their choice of parameters led to the conclusion that the approximation is valid for length to thickness ratio greater or equal to ten.

Han and Lefkowits [3] confirmed that the assumption of uniform heat transfer coefficient is unrealistic. They assumed a power series distribution for the heat transfer coefficient as shown in equation (1), and used it to find the temperature distribution and the fin efficiency for straight fins, i.e.:

$$h(x) = (\gamma + 1) * h_a * \left[\frac{x}{L} \right]^\gamma \quad \dots (1)$$

Where γ a constant, L is the fin length and h_a is the average heat transfer coefficient over the entire fin surface.

Vind, [3], applied a finite difference computer program to find the temperature profiles for annular fins of constant thickness and of tapered profile. The variable heat transfer coefficient distributions used in the investigation are given by:

$$h(R) = 2 * h_a * R \quad \dots (2)$$

and

$$h(R) = 3 * h_a * R^2 \quad \dots (3)$$

$$\text{Where } R = \frac{r - r_i}{r_o - r_i}$$

Higges, [4], studied the heat transfer from annular fins of triangular profile with variable heat transfer coefficient that is increasing from the base to the tip. It was found that the increase in the heat transfer coefficient causes a decrease in the efficiency of the fin.

Unal, [5], studied the effect of variable heat transfer coefficient on the efficiency and the effectiveness for straight fins. The heat transfer coefficient was assumed to depend on the fin surface temperature according to:

$$h = a_1 T^{n_1} \quad \dots (4)$$

Where a_1 and n_1 are constants. It was concluded that the decay in the temperature along the fin led to a reduction in the efficiency and effectiveness.

Imre and Razelos, [6], presented an analytical solution to calculate the heat transfer from annular fins. The assumptions used in the work include one dimensional heat flow, insulated fin tip and non-uniform heat transfer coefficient over the fin surface. The equation used to describe the variable heat transfer coefficient over the fin surface was given by:

$$h(r) = ka \left[\frac{r - r_i}{r_o - r_i} \right]^{am} \quad \dots (5)$$

where am is a constant,

$$ka = \left[\frac{(Rm + 1) * (am + 1) * (am + 2)}{2 * ((am + 1) * Rm + 1)} \right] * ha \quad \dots (6)$$

and $Rm = (r_o/r_i)$

Kahwaji, [7], conducted a numerical and electrical analogue study of the thermal performance of annular fins of constant thickness under the one and two dimensional heat flow assumption. He suggested a new simple method for correlating the fin rate of heat transfer which depends on the grouping of the different thermal resistances of the fin in a (driving force / resistance) form model. The suggested resistance is given as:

$$TR_{th} = \left(\left(\frac{\ln(r_o / r_i)}{2\pi kW} \right)^2 + \left(\frac{W}{4\pi k(r_o^2 - r_i^2)} \right)^2 \right)^{\frac{1}{2}} + \frac{1}{2\pi h(r_o^2 - r_i^2)} \quad \dots(7)$$

Where TR_{th} is the total resistance of the fin and the term under the square root is the two - dimensional material resistance while the other term is the surface resistance. The above form of correlation was found to give an accurate representation of the heat dissipation by the fin when compared to that obtained from the analogue system. Moreover, the assumption of one - dimensional heat flow through the fin was found to be valid when the total resistance is greater than 835°C/kW .

Kahwaji and Al-Makhyoul, [8], conducted detailed experiments to validate the above resistance model experimentally. The experiments covered both the natural and forced convection heat transfer modes using fins of different materials and dimensions. The results indicated a good agreement between the suggested model and the experimental findings. The calculated differences were less than (8.33%) in the natural convection tests and less than (11%) in the forced convection tests. However, a uniform heat transfer coefficient along the fin surface was employed in both of the discussed investigations above, [4 and 5].

In this paper, a method for incorporating variable heat transfer coefficients in the total resistance model will be derived analytically and its validity will be numerically investigated over a wide range of fin parameters and heat transfer coefficient models.

2. Analysis:

Due to the non linear distribution of the circular fin area along its radius, the overall heat conductance can be calculated from:

$$U = \int d(hA) \quad \dots (8)$$

Incorporating the variable heat transfer coefficient models of Emre and Razelos, [6], given by equations (5 and 6), equation (8) above yields:

$$U = \int ka \left[\frac{r - r_i}{r_o - r_i} \right]^{am} * 2\pi r dr \quad \dots (9)$$

for $r_i \leq r \leq r_o$

$$dU = 2\pi ka \left[\frac{r - r_i}{r_o - r_i} \right]^{am} r dr \quad \dots (10)$$

and the surface conductance, for the fin is:

$$U = \frac{2\pi ka [(am + 2)(ro - ri)^{am+1} ro - (ro - ri)^{am+2}]}{(ro - ri)^{am} (am + 1)(am + 2)} \quad \dots (11)$$

substitution of equation (11) in the total resistance for the annular fin, equation (7), yields :

$$TR_{th} = \sqrt{\left[\left(\frac{\ln(r_o / r_i)}{2\pi KW} \right)^2 + \left(\frac{W}{4\pi K(r_o^2 - r_i^2)} \right)^2 \right]} + \frac{(ro - ri)^{am} (am + 1)(am + 2)}{2\pi ka [(am + 2)(ro - ri)^{am+1} ro - (ro - ri)^{am+2}]} \quad \dots (12)$$

Finally the heat dissipation by the fin, according to [6], is given by:

$$QF_{th} = \frac{T_b - T_f}{TR_{th}} \quad \dots (13)$$

3. The numerical solution:

The two-dimensional conduction equation governing the problem is given by, see figure (1):

$$QF_{th} = \frac{T_b - T_f}{TR_{th}} \quad \dots (14)$$

subject to :

$$\frac{\partial T}{\partial r} = \frac{\partial T}{\partial z} = 0.0 \quad \text{at the tip and center line respectively, } T=T_b \text{ at the base and}$$

$$\frac{\partial T}{\partial r} = -\frac{h(r)}{k} (T_s - T_f) \quad \text{along the surface.}$$

Second order central difference formulation is used to discretize equation (16) above and a second order forward or backward formulas are used for the flux terms. These yield:

$$h (R) = 2 * ha * R \quad \dots (15)$$

for the interior points, where:

$$a = (2+\gamma) / (4+4\beta) = (0.5+0.5\gamma) / (1+\beta) \quad \dots (16)$$

$$b = \beta / (2+2\beta) \quad \dots (17)$$

$$c = (0.5-0.25\gamma) / (1+\beta) \quad \dots (18)$$

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$$\gamma_i = (\Delta r/r_i), \beta = (\Delta r/\Delta z)^2 \quad \dots (19)$$

For the fin tip, the formula is:

$$T(N, j) = \left(\frac{1}{2 \in i \Delta r + 3} \right) (-4T(N-1, j) + T(N-2, j)) + \frac{T_f}{1 + \left(\frac{3}{2 \in i \Delta r} \right)} \quad \dots (20)$$

where $\in = h_i/k$

For the fin surface:

$$T(i, M) = \left(\frac{1}{2 \in i \Delta z + 3} \right) (-4T(i, M-1) + T(i, M-2)) + \frac{T_f}{1 + \left(\frac{3}{2 \in i \Delta z} \right)} \quad \dots (21)$$

and for the symmetry line

$$T(i, 1) = \frac{4T(i, 2) - T(i, 3)}{3} \quad \dots (22)$$

In the formulation above, h_i is calculated from equation 5 using different values for the exponent (am). The above system of equations is solved using the SOR technique and the heat dissipation by the fin is obtained from the summation of the flux over the fin surface, i.e.:

$$Q_{Fn} = \sum_{i=1}^{i=N} 4\pi * r(i, M) * \Delta r * h(i) * (T(i, M) - T_f) \quad \dots (23)$$

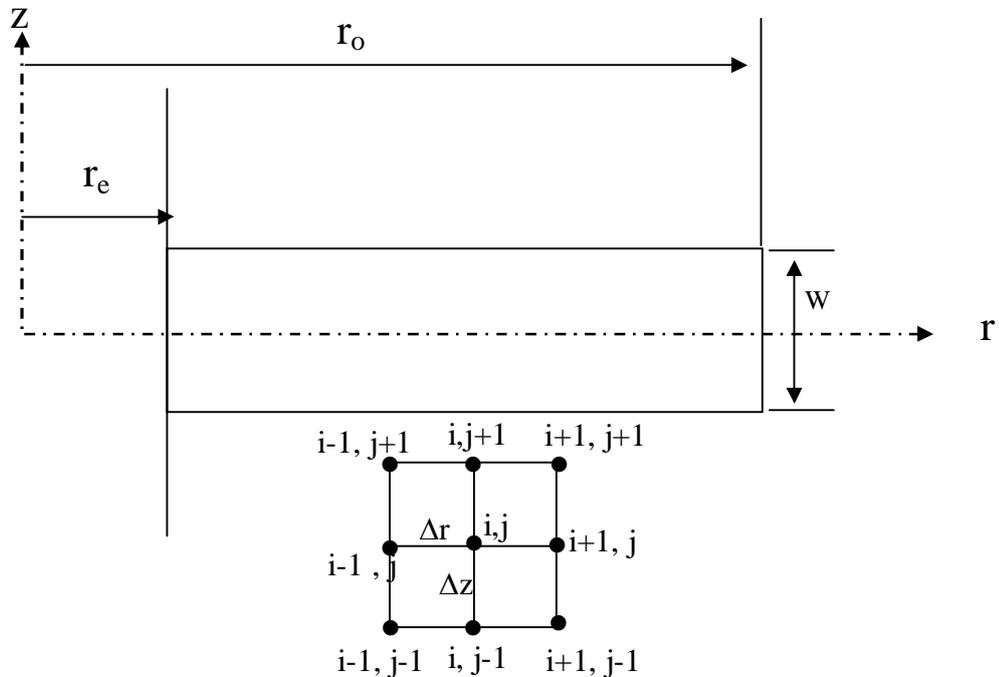


Figure (1) the dimensions of annular fin constant thickness

4.Results and discussion:

Figure (2) shows the temperature distribution along the fin in the radial direction for constant and variable heat transfer coefficient. Near the root, the local temperature gradient is lower for the variable h case due to the higher surface resistance which is in agreement with the experimental findings of many researchers and supports the hypothesis that in real life the heat transfer coefficient on the fin surface is indeed variable. The gradient varies nearly linearly in the case of variable (h) due to the exponential increase the total conductance along the fin as shown in figure (3).

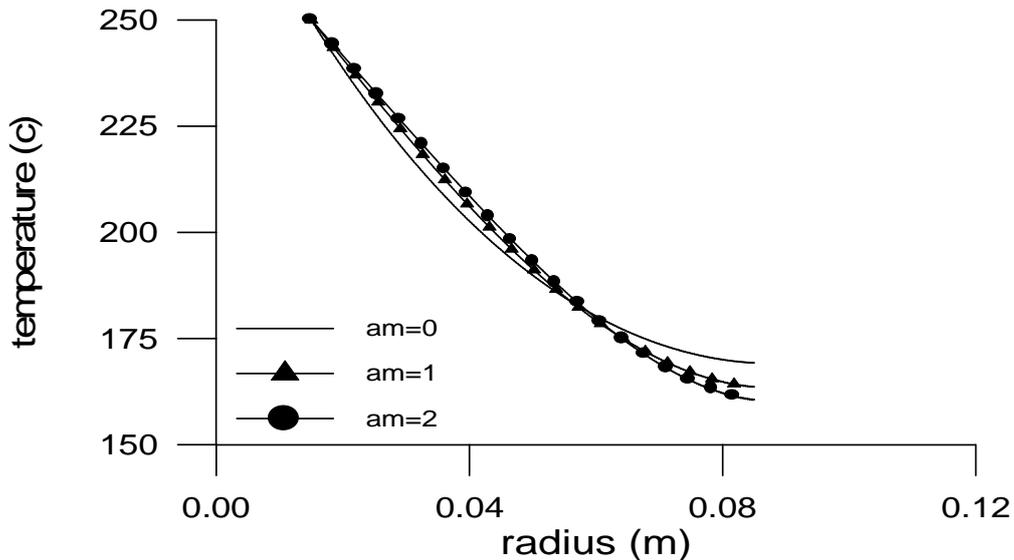


Figure (2) Temperature distribution along the fin

Figure (3) shows the distribution of local surface conductance along the fin using constant heat transfer coefficient with $am=0$ and variable heat transfer coefficient with $am=1$ and 2. Of interest are the non-zero value at the base for the uniform h case and the exponential increase at the tip for the non-uniform h case. Both are believed to be unrealistic considering the hydrodynamics of the flow within the fin passage.

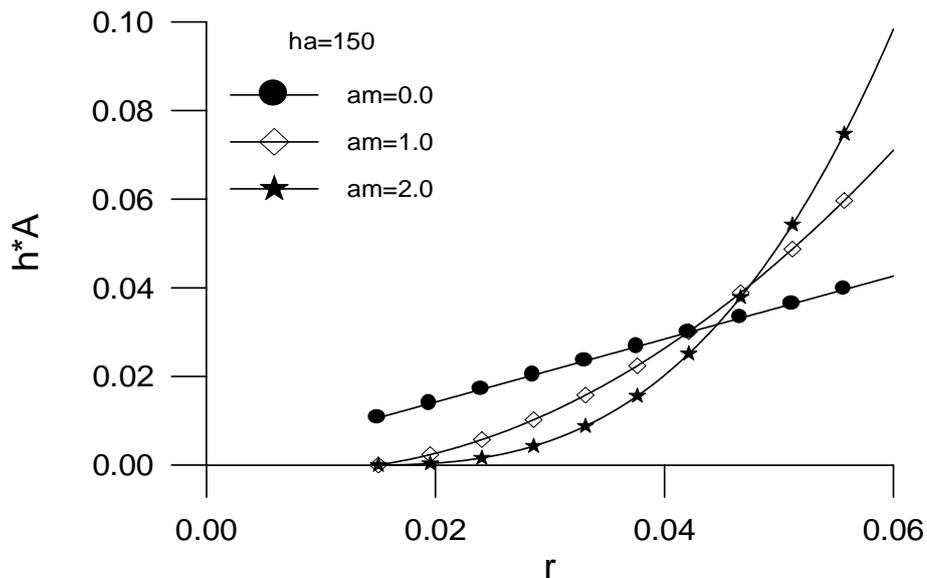


Figure (3) variation of thermal conductance along the fin with different h models

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Figure (4) shows the behavior of heat dissipation along the fin length for the variable and uniform heat transfer coefficient cases. For the uniform coefficient case, close to the base the heat dissipation increases gradually and is higher than the other cases due to the high surface conductance value that offsets the decrease in the temperature gradient. In the variable heat transfer coefficient case, moving along the fin away from the base shows an increase in the heat dissipation; however this increase is lower than that of the uniform coefficient case due to the lower surface conductance along the effective section of the fin length. Increasing the exponent in equation (5) further reduces the heat dissipation. The same results are obtained when varying the fin length as shown in figure (5).

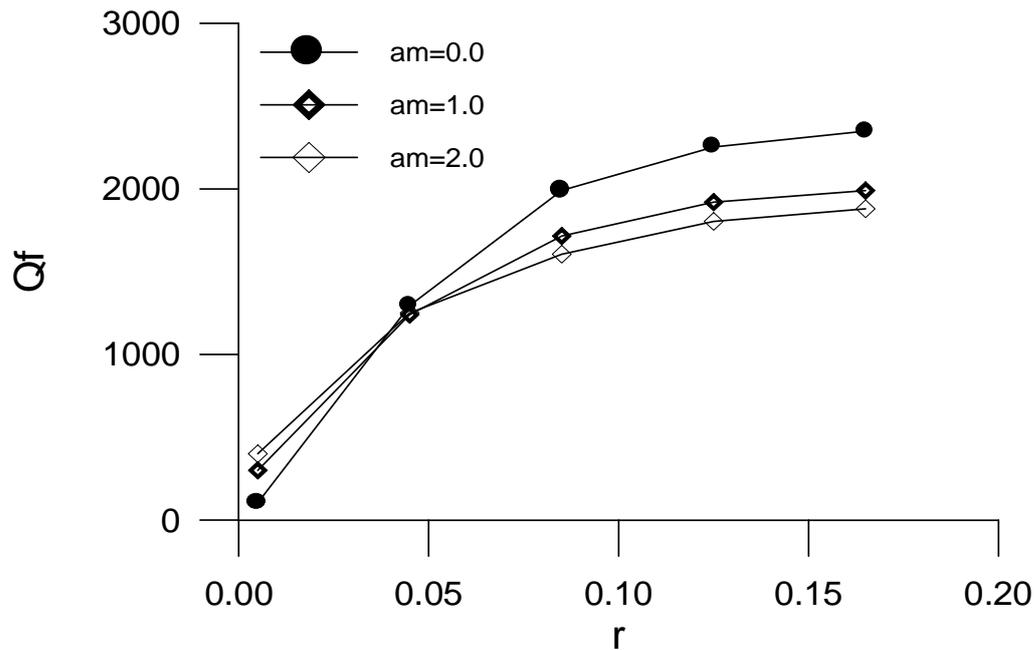


Figure (4) heat dissipation along the fin with constant and variable heat transfer coefficient

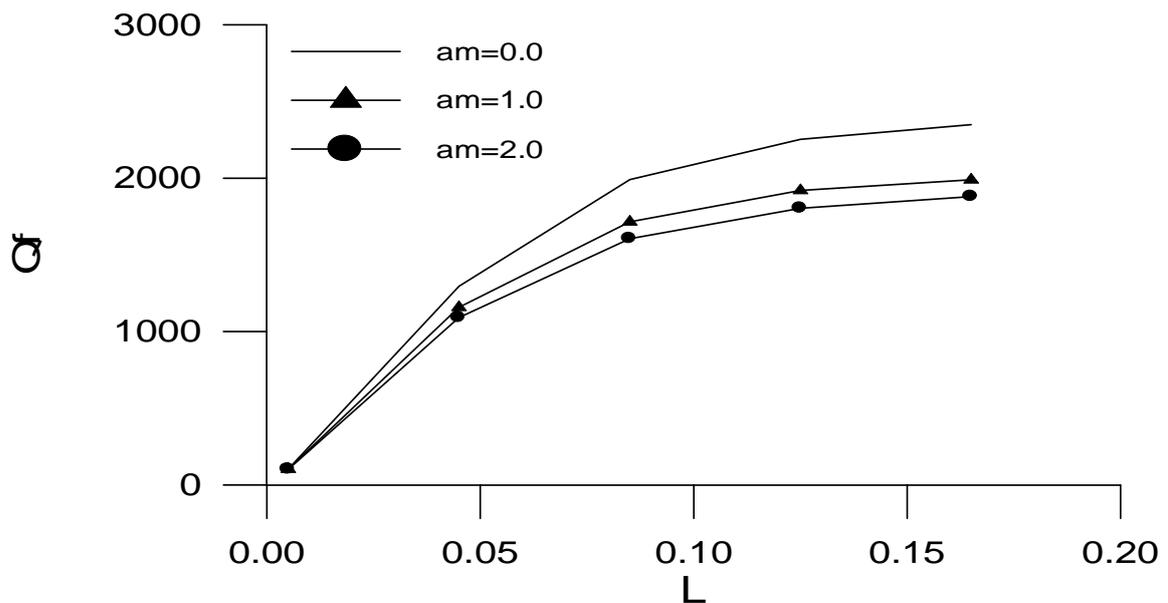


Figure (5): Variation of the heat dissipation with the fin length under various heat transfer coefficient models.

The target of this study was to establish the validity of the total resistance model for heat dissipation through circular fins with variable heat transfer coefficients. Figure (5) shows the results of the analysis conducted for $0.001 < W < 0.005$ m, $0.005 < L_f < 0.10$ m and $5 < h_a < 200$ $W/m^2 \cdot ^\circ C$. All points fall on the same straight line which indicates the sufficiency of the proposed model and its adequacy to explain the phenomenon. The results proved the validity of the suggested total resistance model under the variable heat transfer coefficient assumption and supports the analogue and experimental findings reported by [5] and [6] for the constant heat transfer coefficient assumption. Furthermore, it establishes a method for dealing with the variable heat transfer coefficient case as given by equation 12. It is worth stressing here that the advantage of this model lies in its simplicity and from the fact that it complies with the basic flux- inverse resistance model of any flow, fluid, heat, electricity... Many researches were conducted since the proposal of this model and all of them supported the validity of the model. A review paper is being planned at this time to summarize all the results.

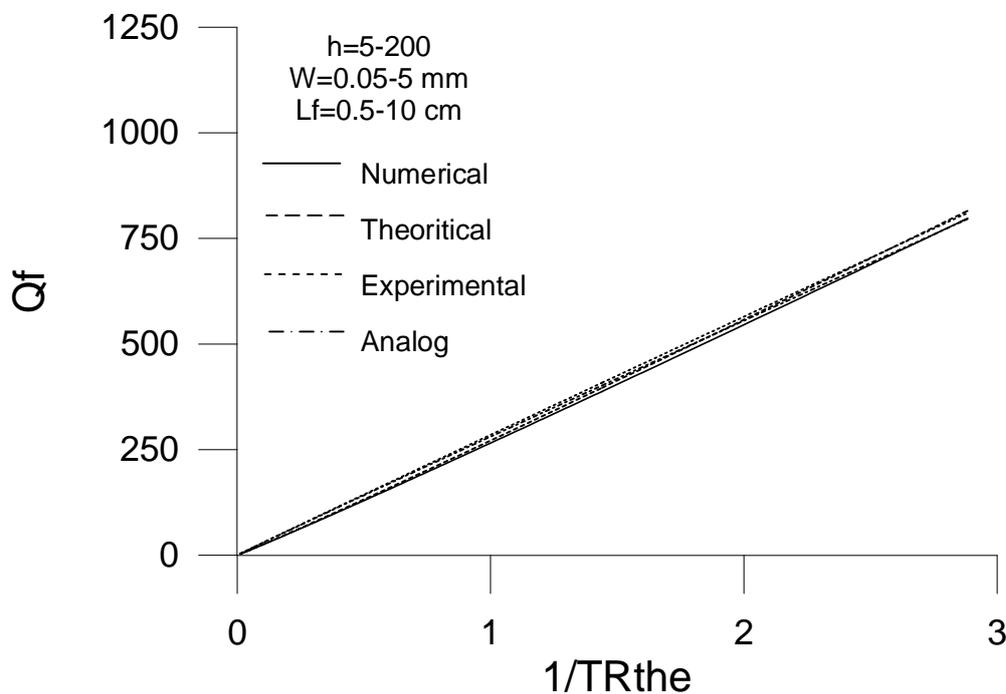


Figure (6) The linear variation of the heat dissipation with the inverse of the total resistance

5. Conclusions:

From this study, a general correlation for heat transfer rate can be concluded with the following limitations:

- 1- Finding a general correlation to calculate the heat transfer rate through annular fins in two dimensions (radial and transverse).
- 2- The correlation is applied only for annular fin with constant thickness
- 3- The correlation can be used for a wide range of fin parameters.
- 4- The correlation is used for either constant or variable distribution of heat transfer coefficient .

6. Nomenclature:

- a,b,c, γ and γ_1 : constants
A: Fin area
Exp: experimental
ha: Average heat transfer coefficient ($\text{W/m}^2 \cdot ^\circ\text{C}$)
h(r): Local heat transfer coefficient ($\text{W/m}^2 \cdot ^\circ\text{C}$)
K: Thermal conductivity ($\text{W/m} \cdot ^\circ\text{C}$)
L: Length of fin (m)
Num: Numerical
r: Local radius of annular fin (m)
ro: Outer radius of fin (m)
ri: Inner radius of fin (m)
 $R=(r-r_i)/(r_o-r_i)$
 $R_m=(r_o/r_i)$
 T_s : Surface temperature ($^\circ\text{C}$)
 T_b : Base temperature ($^\circ\text{C}$)
 T_f : Fluid temperature ($^\circ\text{C}$)
 TR_{th} : theoretical total resistance ($^\circ\text{C/W}$)
QF: The heat transfer from fin (W)
 QF_{th} : The heat transfer from fin theoretically (W)
QFn: The heat transfer from fin numerically (W)
W: Fin thickness
x: The local distance of longitudinal fin (m)

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