An Analytical Design Procedure For Bireciprocal Lattice Wave Digital Filters With Approximate Linear Phase

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Abstract

A simple analytic design procedure for bireciprocal lattice wave digital filters (bireciprocal LWDFs) is presented with approximate linear phase. The design is started by replacing the odd order all-pass filter branch in the bireciprocal LWDF with a pure delay, leaving the other branch as an all-pass even function of z^{-2}. Analytic design procedure is then formulated. Several design examples using such procedure are given for verifications.

Keywords: Bireciprocal LWDFs, All-pass sections, Half-band filters, Approximate linear phase.

أسلوب تحليلي لتصميم مرشحات رقمية موجية من النوع المتشابك ثنائي التبادل وبطور خطي تقريبي

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الخلاصة

روعي في هذا البحث تقديم أسلوبا تحليليا مختصرا ومبسطا لتصميم مرشحات رقمية موجية من النوع المتشابك ثنائي التبادل (bireciprocal LWDFs) وبطور خطي تقريبي. لقد تم تصميم هذه المرشحات باستخدام تجزئة هـذه المرشحات إلى فرعين متوازيين يحتوي ألأول منها على وحدة تأخير خالصة (a pure delay) برتبة فرديـة. بينما يحتوي ألفرع ألثاني على مقاطع إمرار كلية (all-pass sections) برتبة زوجية. أمكن الحصول على استجابات خطية ألطور تقريبية لهذه المرشحات وبأسلوب تصميم مبسط وكما موضح في ألأمثلة.

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I. Introduction

A wave digital filter (WDF) is the digital counterpart of a corresponding analogue filter in the analogue reference domain. This makes the design of WDFs be basically carried out in the analogue domain using classical filter approximations followed by the application of certain analogue to digital transformations rules [1].

Among all other types of recursive filters, wave digital filters are known to have many advantageous properties. They have low coefficient sensitivity, good dynamic range, and especially, good stability properties under finite-arithmetic conditions [1],[2]. Unfortunately, suboptimal design method of those WDFs results in very high complexity implementations. Particularly, favorable wave digital filters are the lattice wave digital filters (LWDFs) [2]. Using lattice WDFs, highly modular and parallel filter algorithms can be obtained. This makes them suitable for VLSI implementations as they have regular low complexity structures, low coefficients sensitivity, and can yield optimal pipelining for bit-serial implementations of maximally high-speeds [3],[4]. Some efficient pipelined WDFs are widely used in wideband high-pass applications such as wireless codec design or ECG signal processing [5]. Some others LWDFs guarantee that the optimum finite-wordlength solution can be found for both fixed-point and multiplierless coefficient representations [6]. Wave digital realizations can also be obtained from the specifications, through VHDL descriptions and then synthesized into Xilinx FPGA implementations [7]. In addition to that, the LWDF is well suited for microcontrollers without a hardware multiplier [8].

Among other LWDF applications, the high-speed integrated circuits and the multirate IF filters for mobile radio using LWDF implementations in silicon may be highlighted [9],[10]. Recently, wavelet transform implementations and wavelet bases are obtained from orthonormal nonseparable perfect reconstruction quadrature mirror filter (QMF) banks that are realized with LWDFs [11]. More recently, LWDF are properly utilized in pulse shaping, audio / image processing systems, digital camera and mobile phones [12], while, glass breakage detectors are simply designed using LWDFs on MSP430 chip [13]. Gazsi in 1985 [14] reported the first design of LWDFs using some explicit formulas for the direct computation of the adaptor coefficients starting from the poles of the transfer function of the analogue filter predesigned by classical filter approximation techniques. Such design uses the alternative pole technique for the realization of the WDF composed of two all-pass filter sections in parallel. However, such LWDFs can only satisfy some magnitude requirements without taking any phase requirement into considerations. Most attempts to design such LWDFs satisfying both magnitude and phase requirements face the problem of no closed form solutions existence. For those attempts, numerical optimization techniques must be adopted [1].

The idea of LWDFs is then extended to the design of almost linear phase LWDFs by replacing one of the two parallel all-pass sections by a pure delay [15]. Bireciprocal LWDF structures are preferred over LWDF ones because of their less complexity and minimal time delay. Nevertheless, their versions with approximate phase linearity still face the complexity of the design techniques [16].

In this paper, a simple analytic design procedure for bireciprocal LWDFs with approximate linear phase is presented. The procedure is based on the prescribed idea of letting one of the two parallel all-pass filter sections be a pure delay to result in an almost linear phase LWDFs. Section II of this paper presents the basic ideas of LWDFs and bireciprocal LWDFs. The bireciprocal LWDFs with almost linear phase are described in section III. The design procedure is presented in section IV. Section V contains several design examples. Finally, section VI concludes this paper.

II. Lwdfs And Bireciprocal Lwdfs

An LWDF is, as shown in Fig. 1, a two-branch structure where each branch realizes an all-pass filter [14]-[16]. These all-pass filters can be realized in several ways. One approach that yields parallel and modular filter algorithms is to use cascaded first- and second-order sections. The first- and second-order sections can be realized using symmetric two-port adaptors [1],[15] (see Fig. 2). Two-port series or parallel adaptors using certain equivalence transformations can easily replace these sections. The second-order sections can also be realized using three-port series or parallel adaptors [15]. Another approach is to realize the all-pass filters using Richard's structure [1], where a processing element can easily be formed to accomplish a bit-serial low-power implementation with low-complexity.

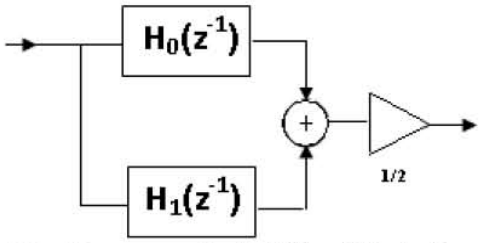

Fig.1 Lattice wave digital filter block diagram.

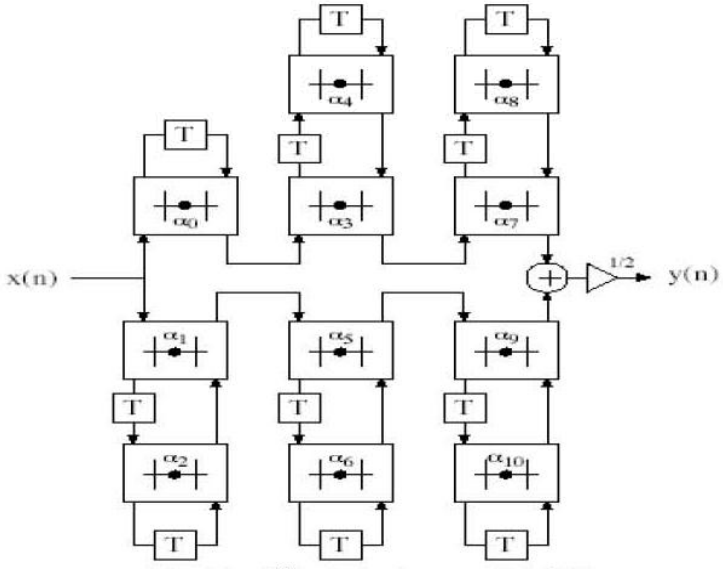

Fig. 2 An 11th order lattice wave digital filter.

$$H(z) = \frac{1}{2}\,[\,H_0(z) + H_1(z)\,] \qquad (1)$$

where $H_0(z)$ and $H_1(z)$ are all-pass filters. The overall frequency response can therefore, be written as :

$$H(e^{j\omega T}) = \frac{1}{2}\,[\,e^{j\Phi_0(\omega T)} + e^{j\Phi_1(\omega T)}\,] \qquad (2)$$

where $\Phi_0(\omega T)$ and $\Phi_1(\omega T)$ are the phase responses of $H_0(z)$ and $H_1(z)$, respectively. The magnitude of the overall filter is thus limited by

$$|H(e^{j\omega T})| \leq 1 \qquad (3)$$

The transfer function of a LWDF and its complementary transfer function are power complementary, i.e.,

$$|H(e^{j\omega T})|^2 + |H_c(e^{j\omega T})|^2 = 1 \qquad (4)$$

where

$$H_c(z) = \frac{1}{2}\,[\,H_0(z) - H_1(z)\,] \qquad (5)$$

This means that, if H(z), for example, is a low-pass filter, then a high-pass filter $H_c(z)$ can be obtained by simply changing the sign of the all-pass filter $H_1(z)$ in (1). It is known that, an attenuation zero exists corresponding to an angle $\omega_0 T$ at which the magnitude function reaches its maximum value. For LWDFs, this occurs when [15]

$$|H(e^{j\omega_0 T})| = 1 \qquad (6)$$

A transmission zero exists corresponding to an angle $\omega_1 T$ at which the magnitude function is zero, i.e. when

$$|H(e^{j\omega_1 T})| = 0 \qquad (7)$$

At an attenuation zero, the phase responses of the branches must take the same value. Hence, in the pass-band of the filter, the phase responses must be approximately equal , i.e.

$$\Phi_0(\omega T) = \Phi_1(\omega T) \qquad (8)$$

while, at a transmission zero, the difference in phase between the two branches must be

$$\Phi_0(\omega T) - \Phi_1(\omega T) = \pm\pi \qquad (9)$$

Thus, the difference in phase between the two branches must approximate $\pm\pi$ in the stop-band of the filter. To make sure that only one pass-band and one stop-band occur, the orders of $H_0(z)$ and $H_1(z)$ must differ by one [15],[16].

In terms of computational effort, bireciprocal LWDFs represent the most efficient family of IIR filters and are therefore of great interest. It is therefore very important to design linear-phase bireciprocal LWDFs to obtain efficient structures preserving the phase linearity property. A bireciprocal (half-band) LWDF is a special case of LWDF. In this case every other coefficient of the filter becomes 0, which results in a reduced structure such as the one shown in Fig. 3. Moreover, when the application is in a decimator or interpolator by a factor of 2, the filter can run at the lower sampling rate [17]. The transfer function of a bireciprocal LWDF can be written as

$$H(z) = \frac{1}{2}\,[\,H_0(z^2) + z^{-1} H_1(z^2)\,] \qquad (10)$$

where the transfer function $H_0(z^2)$ corresponds to the lower branch in Fig. 3. The transfer function of the filter and its complementary transfer function are power complementary. Therefore, for bireciprocal LWDFs

$$\left|H(e^{j\omega T})\right|^2 + \left|H(e^{j(\omega T-\pi)})\right|^2 = 1 \qquad (11)$$

which means that the pass-band and stop-band edges are related by $\omega_c T + \omega_s T = \pi$ with ω_c and ω_s being respectively, the pass-band and stop-band cutoff frequencies. The consequence is that the pass-band ripple will be extremely small for practical requirements on the stop-band attenuation. Thus the bireciprocal WDFs have the efficiency of an FIR half-band filter in terms of reduced computational effort (compared to non half-band counterparts), while preserving the main advantages of IIR filters over FIR, i. e., sharp transitions for lower orders. Moreover, it is a well-known fact, that WDFs have very low multiplier coefficients sensitivity. Thus it is possible to represent filter coefficients utilizing only a few bits. This could allow for decreasing the size of applied multipliers or even replacing them by shift and add operations. All algorithms, previously used to design bireciprocal LWDFs utilize numerical optimization methods.

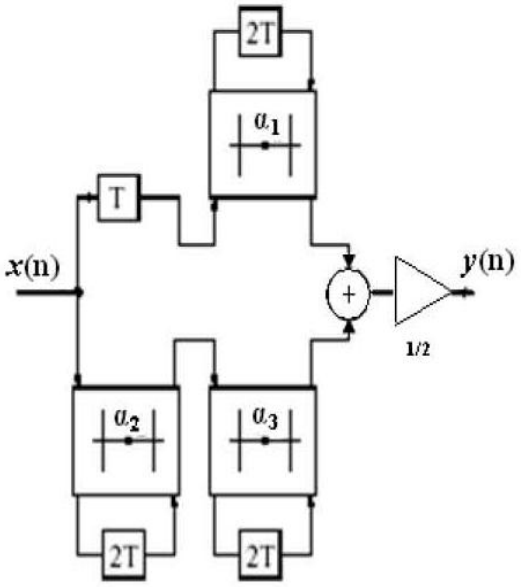

Fig. 3 A 7th-order bireciprocal lattice wave digital filter.

III. Almost Linear Phase Bireciprocal Lwdfs

It is possible to obtain a bireciprocal LWDF with approximate linear phase by letting one of the branches in Fig. 3 consist of a pure $(2R-1)^{th}$ order delay [4],[15] (see Fig. 4). The other branch $H_o(z^{-2})$ (of even order 2N) is a general all-pass function in z^2, which can be realized using cascaded first and second orders sections. The transfer function of a linear phase bireciprocal LWDF is

$$H(z) = \frac{1}{2}\left[H_0(z^{-2}) + z^{-(2R+1)} \right] \qquad (12)$$

$$H_0(e^{-j2\omega T}) = \frac{\sum_{n=0,even}^{2N} b_n e^{-jn\omega T}}{\sum_{n=0,even}^{2N} b_n e^{-j(N-n)\omega T}} \quad (18)$$

The phase response of this all-pass branch can be reduced to

$$\Phi_0(\omega T) = 2\tan^{-1}\left(\frac{\sum_{n=0,even}^{2N} bn \sin n\omega T}{\sum_{n=0,even}^{2N} bn \cos n\omega T}\right) \quad (19)$$

or

$$\sum_{n=0,even}^{2N} bn \sin n\omega T = \tan\left(\frac{\Phi_0(\omega T)}{2}\right) \sum_{n=0,even}^{2N} bn \cos n\omega T$$

i.e.,

$$\sum_{n=0,even}^{2N} bn \left[\sin n\omega T - \tan\left(\frac{\Phi_0(\omega T)}{2}\right) \cos n\omega T\right] = 0 \quad (20)$$

According to (15) and (16), (20) can be formulated in the pass-band as follows:

$$\sum_{n=0,even}^{2N} bn \left[\sin n\omega T - \tan\left(\frac{-(2R+1)\omega T}{2}\right) \cos n\omega T\right] = \pm\delta, \quad \text{for } (0 \le \omega T \le \omega_c T) \quad (21)$$

and in the stop-band as follows:

$$\sum_{n=0,even}^{2N} bn \left[\sin n\omega T - \tan\left(\frac{-(2R+1)\omega T - \pi}{2}\right) \cos n\omega T\right] = \pm\delta, \quad \text{for } (\omega_c T \le \omega T \le \pi) \quad (22)$$

where $\delta \ll 1$.

By selecting (N+1) extremal points on the union of the pass-band and the stop-band regions. Therefore, (21) and (22) are sampled in these frequency points, while proper alternating $\pm\delta$ values are examined at these points. One can start with an initial point $\omega_1 T > 0$ and the other points in the pass-band and stop-band can then be distributed equidistantly in the rest band. In matrix form, we can write the sampled version of (21) and (22) as

$$A\,B = \delta \quad (23)$$

where A is an (N+1) x (N+1) matrix given by

$$A = [a_{ij}], \; i, j = 1, 2, 3, \ldots\ldots (N+1) \quad (24)$$

B is an (N+1) x 1 matrix, having only even order coefficients b_n's , since all other odd coefficients are zeros. B can be written in a transposed form as

$$B^t = [b_0 \quad b_2 \quad b_4 \quad . \quad . \quad . \quad b_{2N}] \quad (25)$$

and δ an (N+1) x 1 matrix can be written in a transposed form as

$$\delta^t = [\delta \quad -\delta \quad \delta \quad -\delta \quad . \quad . \quad . \quad \delta] \quad (26)$$

with

$$a_{ij} = \sin 2(j-1)\omega_i T - \tan\left(\frac{-(2R+1)\omega_i T}{2}\right) \cos 2(j-1)\omega_i T \quad (27)$$

in the passband ($0 < \omega_i T \leq \omega_c T$), i =1, 2, 3,…., N/2 (N even) [or i =1, 2, 3, ….,(N+1)/2 (N odd)] and j =1, 2, 3,….., N+1. and

$$a_{ij} = \sin 2(j-1)\omega_i T - \tan\left(\frac{-(2R+1)\omega_i T - \pi}{2}\right) \cos 2(j-1)\omega_i T \qquad (28)$$

in the stop-band ($\omega_c T < \omega_i T \leq \pi$), i = (N/2) + 1, (N/2) + 2, ….., N+1 (N even) [or i = [(N+1)/2] +1, [(N+1)/2] + 2, ….., N+1 (N odd)] and j = 1, 2, 3….., N+1. In this analytic algorithm, δ can be selected properly to solve

$$B = A^{-1}\ \delta \qquad (29)$$

Thus, the design algorithm of the linear-phase bireciprocal LWDF is reduced now to the evaluation of the vector $B^t = [b_0 \quad b_2 \quad b_4 \quad . \quad . \quad . \quad b_{2N}]$ which represents all the b_n's even coefficients that should appear in the all-pass function $H_0(z^{-2})$ of (17), while the total $H_{LPF}(z^{-1})$ is the one given in (12). Polynomial factorization can be used to expand $H_0(z^{-2})$ of (17) into a product of 2nd order all-pass sections, while all the multiplier coefficients α_i of the corresponding 2nd order adaptors in branch $H_0(z^{-2})$ can then be evaluated by using the same methods
given in [18].

It should be noted, here, that to design the corresponding high-pass complement filter, one can change the plus sign to minus in (12), i.e., change the adder in Fig. 4 to a subtractor, to find the total high-pass function $H_{HPF}(z^{-1})$. It should also be noted that low-pass LWDF can be transformed to a band-pass one $H_{BPF}(z^{-1})$ by setting each z in $H_{LPF}(z^{-1})$ equal to $-z^2$.

V. Design Examples

Four different examples are illustrated in this section to examine the above design procedure of approximately linear phase half-band bireciprocal LWDFs, using the algorithm described in section IV. They are a 7th order LPF, another 7th order HPF, an 11th order LPF, and another 11th order HPF. The design procedure is applied with $\delta = 0.01$. In all these LWDFs, the upper all-pass branches in the bireciprocal LWDF of Fig. 4 will be as follows: a 4th order type (2N = 4), (i. e., R = 1 in the lower branch) for the 7th order LPF, another 4th order type (2N = 4), (i. e., R = 1 in the lower branch) for the 7th order HPF, a 6th order type (2N = 6),(i. e., R = 2 in the lower branch) for the 11th order LPF, and another 6th order type (2N = 6),(i. e., R = 2, in the lower branch) for the 11th order HPF. The resulting $H_0(z^{-2})$ and $H_1(z^{-1})$ with the total low-pass or high-pass functions ($H_{LPF}(z^{-1})$ or $H_{HPF}(z^{-1})$), those correspond the four examples are given in Table-1. The magnitude and phase responses of all these bireciprocal LWDFs are shown in Figs. 5(a & b) - 8(a & b).

Table-1 The resulting $H_o(z^{-1})$ and $H_1(z^{-2})$ with the total functions ($H_{LPF}(z^{-1})$ or $H_{HPF}(z^{-1})$) according to bireciprocal LWDF type.

Type of approximately linear phase bireciprocal LWDF	$H_o(z^{-2})$	$H_1(z^{-1})$	$H_{LPF}(z^{-1})$ or $H_{HPF}(z^{-1})$
7^{th} order LPF with cutoff frequency (0.5π)	$\frac{-0.4608 + 0.3377 z^{-2} + z^{-4}}{1+0.3377 z^{-2} - 0.4608 z^{-4}}$	z^{-3}	$\frac{-0.2304 + 0.1689z^{-2} + 0.5000z^{-3} + 0.5000 z^{-4} + 0.1689 z^{-5} - 0.2304 z^{-7}}{1 + 0.3377 z^{-2} - 0.4608 z^{-4}}$
7^{th} order HPF with cutoff frequency (0.5π)	$\frac{-0.4608 + 0.3377 z^{-2} + z^{-4}}{1+0.3377 z^{-2} - 0.4608 z^{-4}}$	z^{-3}	$\frac{-0.2304 + 0.1689z^{-2} - 0.5000 z^{-3} + 0.5000 z^{-4} - 0.1689 z^{-5} + 0.2304 z^{-7}}{1+0.3377 z^{-2} - 0.4608 z^{-4}}$
11^{th} order LPF with cutoff frequency (0.5π)	$\frac{0.0043 - 0.2303 z^{-2} + 0.3938 z^{-4} + z^{-6}}{1 + 0.3938 z^{-2} - 0.2303z^{-4} + 0.0043 z^{-6}}$	z^{-5}	$\frac{0.0021 - 0.1151 z^{-2} + 0.1969 z^{-4} + 0.5000z^{-5} + 0.5000z^{-6} + 0.1969 z^{-7} - 0.1151 z^{-9} + 0.0021z^{-11}}{1 + 0.3938 z^{-2} - 0.2303z^{-4} + 0.0043 z^{-6}}$
11^{th} order HPF with cutoff frequency (0.5π)	$\frac{0.0043 - 0.2303 z^{-2} + 0.3938 z^{-4} + z^{-6}}{1 + 0.3938 z^{-2} - 0.2303z^{-4} + 0.0043 z^{-6}}$	z^{-5}	$\frac{0.0021 - 0.1151z^{-2} + 0.1969z^{-4} - 0.5000z^{-5} + 0.5000z^{-6} - 0.1969z^{-7} + 0.1151z^{-9} - 0.0021z^{-11}}{1 + 0.3938 z^{-2} - 0.2303z^{-4} + 0.0043 z^{-6}}$

Vi. Conclusions

A simple design of almost linear phase bireciprocal LWDFs has been presented in this paper. Linear phase responses can approximately be achieved for these structures by replacing one of the all-pass branches of the original structures by a delay unit. Since there exist no closed form solutions for the design of linear phase bireciprocal LWDFs, therefore, numerical optimization algorithms have always been used.. A simple analytic design procedure of almost linear phase LWDFs has been presented in this paper with some examples.

It has been noticed that the magnitude and phase responses of the designed filters give better representations of the desired ones as the order of the bireciprocal LWDF increases (about 85% of the pass-band preserve the linear phase property for filters with order 11). That means more implementation complexity is required for good phase linearity approximations. In spite of that, a single chip implementation using bit-serial, bit-parallel processing elements or FPGA structures may easily be achieved for such bireciprocal LWDF. It is promising topics to use such half-band orthonormal structures in the wavelet transform implementations on a single FPGA chip or to use them as wavelet bases with perfect reconstruction quadrature mirror filter (QMF) banks.

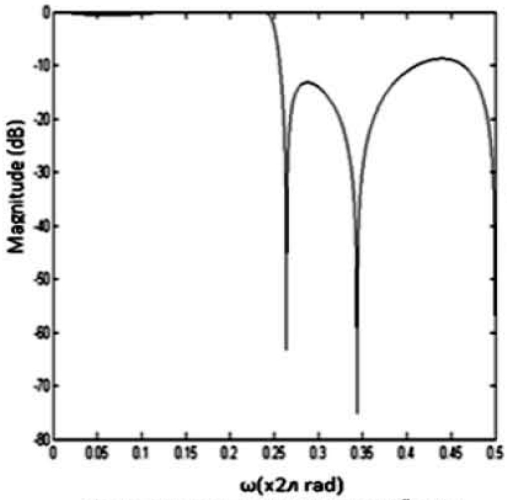

Fig. 5 a) The magnitude response of 7th order bireciprocal LWDF (LPF).

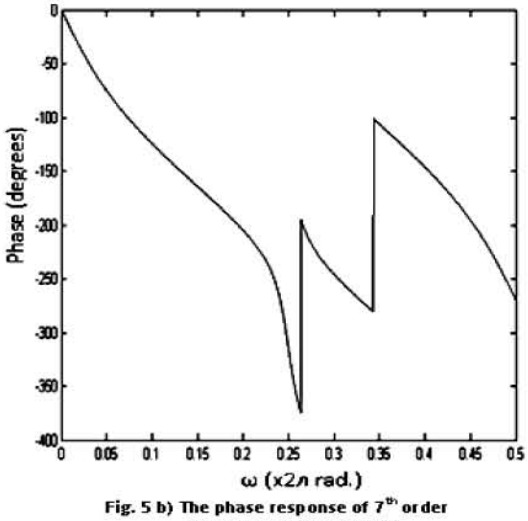

Fig. 5 b) The phase response of 7th order bireciprocal LWDF (LPF).

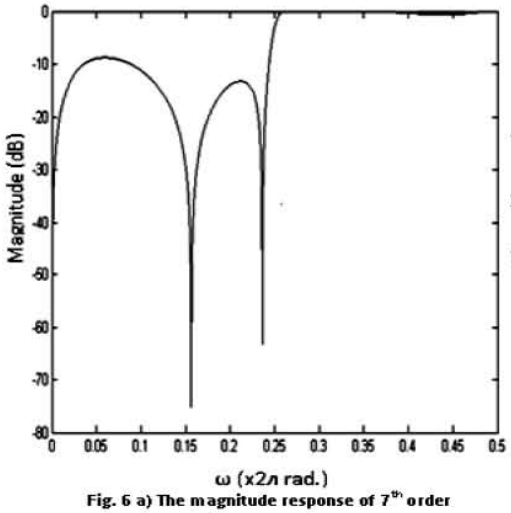

Fig. 6 a) The magnitude response of 7th order bireciprocal LWDF (HPF).

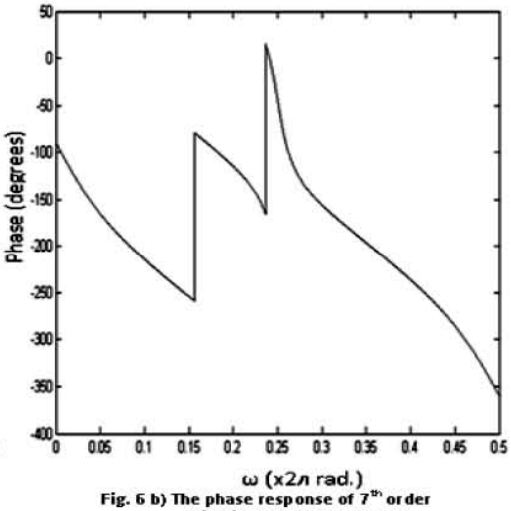

Fig. 6 b) The phase response of 7th order bireciprocal LWDF (HPF).

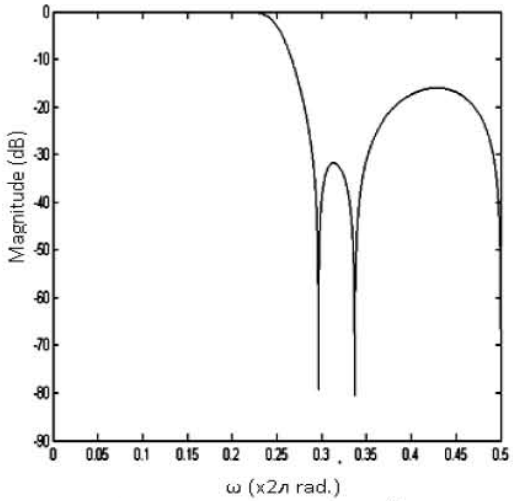

Fig. 7 a) The magnitude response of 11th order bireciprocal LWDF (LPF)

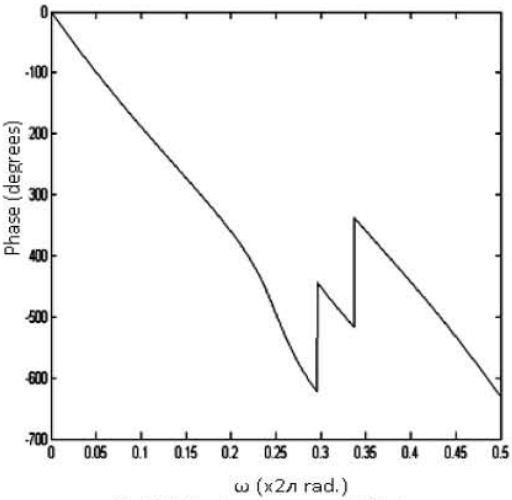

Fig. 7 b) The phase response of 11th order bireciprocal LWDF (LPF)

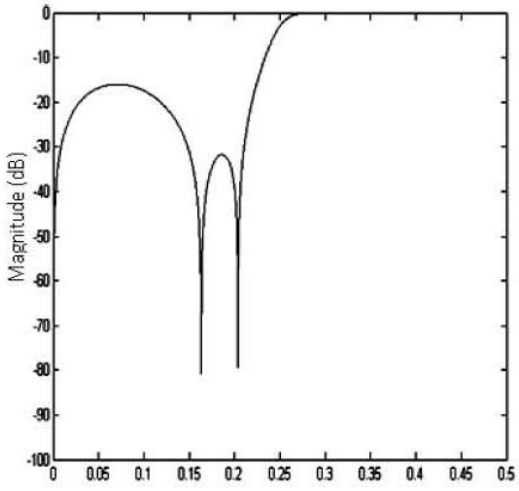

Fig. 8 a) The magnitude response of 11th order bireciprocal LWDF (HPF)

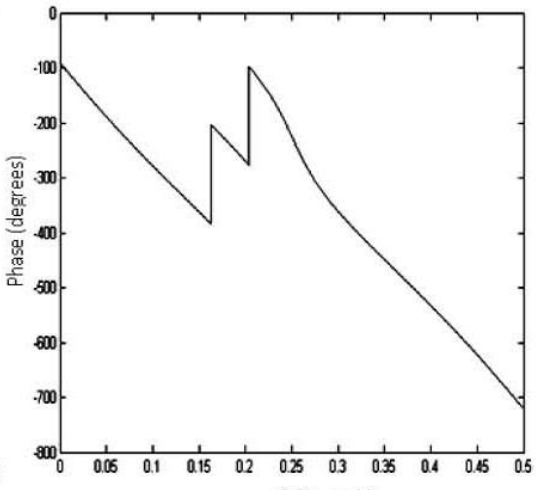

Fig. 8 b) The phase response of 11th order bireciprocal LWDF (HPF)

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