Stability Analysis of MOSUL Dam under Saturated and Unsaturated Soil Conditions

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Abstract

Failure of earth dams can be caused also by seepage problems, foundation instability, deformation, surface sliding, and earthquakes. The most critical conditions that may cause failure of the embankment are: differential settlement, development of shearing within the embankment and foundation, and development of seepage through the embankment and foundation. The stability and factor of safety against Mosul dam embankment sliding are determined considering a possible rapid drawdown and earthquake conditions and using three methods. Unsaturated condition was considered assuming the shear strength parameter (ϕ_b) to be ($0, 0.5\phi, \phi$). GEO-SLOPE OFFICE was used as the analytical tool to simulate both seepage, slope stability, and earthquake. Seepage through dam was analyzed for three period rapid drawdown of water level (30,21,8 day) with the associated saturated-unsaturated transient seepage.

The main results indicated that the minimum slope stability factors of safety were reached using Bishop method and was achieved during 8 day water drawdown and within the second day which indicates the most critical case.

Keywords: Seepage, MOSUL Dam, Finite Element, Slope Stability, Rapid Drawdown, unsaturated soil mechanics.

تحليل استقرارية سد الموصل لحالتي التربة المشبعة وغير المشبعة

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الخلاصة

فشل السدود الترابية يمكن أن يكون نتيجة لمشاكل التسرب وعدم ثبوتية الأسس بالإضافة إلى انزلاق السطح بسبب الهزة الأرضية. إن أكثر الظروف الحرجة التي قد تسبب انهيار المنشأ الترابي هي: الهبوط التفاضلي، نشوء قوى قص خلال الاملائيات والأسس، ونشوء وظهور الجريان خلال الاملائيات والأسس. تم إيجاد ثبوتية وعامل الأمان لاملائيات سد الموصل بالأخذ بنظر الاعتبار أمكانية حدوث تفريغ سريع للمياه أو هزة أرضية وباستخدام عدد من طرق التحليل. اخذ بنظر الاعتبار في التحليل الحالة المشبعة والغير المشبعة لتربة لباب السد وبفرض معاملات القص الغير المشبعة(ϕ_b) مساوية إلى ($0,0.5\phi,\phi$) ، استخدم برنامج GEO-SLOPE OFFICE في التحليل. تم تحليل التسرب خلال السد الترابي بثلاث احتمالات لتفريغ المياه خلال (30,21,8) يوما ومن خلال التحليل الانتقالي للتسرب خلال الترب المشبعة وغير المشبعة.

أظهرت أهم النتائج إن الحد الأدنى لمعاملات ثبوتية المنحدرات التي تم استنتاجها من خلال استخدام طريقتي (Bishop & Lowe-Karafiath). كما تم استنتاج أن الحالة الحرجة كانت خلال ظروف التفريغ السريع للمياه خلال (8) أيام وخلال اليوم الثاني.

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1. INTRODUCTION

Practicing engineering now is well aware that many of the problems they encounter in geotechnical engineering and construction involve unsaturated soils. Most of the engineering problem that involving heave, consolidation, collapse, and dramatic change in shear strength are directly related to the behavior of unsaturated soils.

Numerical simulations represent a good tool that always lead to simplify the real structure and extensively used nowadays. The numerical analysis was also able to qualitatively simulate the behavior and any possible damage pattern of a dam[1]. Sakamoto et al. used numerical simulation to study the sliding of an earth dam during the 1995 Kobe Earthquake, which caused shallow sliding on the upstream slope below water level of Kitayama dam. Results showed that a large residual deformation and shear strain occurred at the shallow area of the upstream slope below the water level.[1]

Many other studies were conducted applying finite element programs (FLAC, SEEP/W and GEO-SLOPE-6) to study the seepage, liquefaction and failure analysis of several earth embankment dams namely, Mochikosk, Merah, Bukit and Labong dams. Results were compared with the real observation of same dams (Byrne and Seid-Karbasei, Kaadan et. al., Mohammed et. al., Kerkes, et. al., Chen and Zhang). Results indicated a good ability of the mathematical models to describe, simulate, analyze and predict many dynamic and hydraulic properties of the studied earth dams.[2,3,4,5,6]

In common practice, it is normal for designer to provide an appropriate factor of safety in their analysis of slope stability. This is important to make sure that the designed slopes are safe and to prevent critical condition where the stress mobilized in soil is more than or equal to the shear resistance and to prevent any unexpected factors during analysis and construction such as wrong data, analysis mistakes, poor workman ship and supervising at sik, etc…. . Table (1) shows the significance of factor of safety for design [7,8].

Table (1): Significance of Factor of Safety for Design of Slopes

Safety factor	Significance
Less than 1	Unsafe
1.0-1.2	Questionable safety
1.3-1.4	Satis factor for custs, fills, questionable for dam
1.5-1.75	Safe for dam

In this study the effect of negative pore water pressure in the driest and wettest conditions on each of seepage analysis, slope stability analysis, and quake analysis is studied using computer modeling software (GEO-SLOPE-5). This work is focused on the stability of Mosul dam considering a possible rapid drawdown and earthquake conditions. The possible enforce rapid drawdown due to water evacuation from reservoir (in case of emergency) will be studied considering three conditions "According to the river valley capacity down stream of the dam and the duration of evacuation of water from the reservoir". The studied conditions are (i): normal condition within 30 days (no risk). (ii): critical condition within 21 days (with some losses). and (iii): urgent condition 8 days water evacuation time.

A finite element analysis using SEEP/W and QUAK/W is accomplished in two steps: model the problem by, designing the finite element mesh, defining the material properties and specifying the boundary conditions followed by analyzing the model by formulating and solving the finite element equations.

2. THEORY

3.1 Seepage Analysis: SEEP/W is formulated on the basis that the flow of water through both saturated and unsaturated soil follows Darcy's low which states that:[9]

$$q = k\,i \qquad \text{..................(1)}$$

Where: q is specific discharge; k is hydraulic conductivity; i is gradient of fluid head.

Darcy's low was originally derived for saturated soil, but later researches [10] has shown that it can also be applied to the flow of water through unsaturated soil.

The governing differential equation used in the formulation of SEEP/W is:[9]

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial H}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial H}{\partial y}\right) + Q = \frac{\partial \theta}{\partial t} \qquad \text{..................(2)}$$

Where H is total heads k_x & k_y are hydraulic conductivity; Q is applied boundary flux; θ is volumetric water content; t time.

Under steady-state condition the equation reduces to:

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial H}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial H}{\partial y}\right) + Q = 0 \qquad \text{..................(3)}$$

The stress state for both saturated and unsaturated condition can be described by two variables. These stress state variables are pore-air pressure ($-u_a$) and matric suction (u_a-u_w) where; u_a is the pore-air pressure, and u_w is the pore-water pressure.

The second assumption is that the pore-air pressure remains constant at atmospheric pressure during transient processes. This means that the formulated in terms of effective stress (σ-u_w) remains constant and has no effect on the change in volumetric water content. Changes in volumetric water content are consequently depend only on the (u_a-u_w) stress state variable, and with u_a remaining constant, the change in volumetric water content is a function only of pore-water pressure changes.

A change in volumetric water content can be related to a change in of pore-water pressure by the equation:

$$\partial \theta = m_w \; \partial u_w \qquad \text{..................(4)}$$

where m_w is the slope of the storage curve.

The total hydraulic head is defined as:

$$H = \frac{u_w}{\gamma_w} + y \qquad \text{..................(5)}$$

where u_w is the pore-water pressure; γ_w is unit weight of water; y is elevation.

u_w=(H-y), by substitution into Equation (2)

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial H}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial H}{\partial y}\right) + Q = m_w\,\gamma_w \frac{\partial (H-y)}{\partial t} \qquad \text{..................(6)}$$

Since the element is constant:

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial H}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial H}{\partial y}\right) + Q = m_w \gamma_w \frac{\partial H}{\partial t} \quad \text{..................(7)}$$

The finite element equation that follows from applying the Galerkin method of weighed residual to the governing differential equation is:

$$\left.\begin{aligned} &\int_v ([B]^T[C][B])dv\{H\} + \int_v (\lambda < N >^T < N >)dv\,(H),t \\ &= q\int_A (< N >^T)dA \end{aligned}\right\} \quad \text{............(8)}$$

where [B] is gradient matrix; [C] is element hydraulic conductivity matrix; {H} is vector of nodal heads.

The finite element solution for a transient analysis is a function of time as indicated by the {H}. The time integration can be performed by a finite difference approximation. writing the finite element equation in terms of finite difference leads to the following equation.

$$(\omega\Delta t\,[K] + [M])\{H\} = \Delta t\left((1-\omega)\{Q_\circ\}\right) + ([M] - (1-\omega)\Delta t[K])\{H_\circ\} \quad \text{...(9)}$$

where Δt is time increment; ω is a ratio between 0 and 1, $\{H_1\}$ is head at end of time increment; $\{H_o\}$ head at start of time increment; $\{Q_1\}$ nodal flux at end of time increment; $\{Q_o\}$ nodal flux at start of time increment; [K] element characteristic matrix; [M] element mass matrix.

Using Gauss numerical integration to form the element characteristic matrix [K] and the mass matrix [M]. The integrals are sampled at specifically defined points in the elements and the summed for all the points.
The following integral:

$$t\int_A ([B]^T[C][B])dA \quad \text{..................(10)}$$

Can be replaced by:

$$\sum_{j=1}^{n}[B_j]^T[C_j][B_j]\,det|J_j|\,W_{1j}W_{2j} \quad \text{..................(11)}$$

where j integration point; n number of integration points; $det|J_j|$ determinant of the Jacobian matrix; W_{1j}, W_{2j} =weighting factors.

3.2 Slope Analysis: SLOPE/W uses the theory of limit equilibrium of forces and moments to computer the factor of safety against failure. A factor of safety is defined as that factor by which the shear strength of the soil must be reduced in order to bring the mass of soil into a state of limiting equilibrium along a selected slip surface. For an effective stress analysis, the shear strength is defined as:[9]

$$s = c' + (\sigma_n - u)\,tan\phi' \quad \text{..................(12)}$$

Where S shear strength; c' effective cohesion; ϕ' effective angle of internal friction; σ_n total normal stress; u pore-water pressure.

For the total stress analysis, the strength parameter are defend in terms of total stresses and pore-water pressures are not required.

Factor of Safety for Saturated and Unsaturated Soil: For unsaturated soil, a modified Mohr-Coulomb equation must be used to described the shear strength of an unsaturated soil:

$$s = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b \qquad \text{..................(15)}$$

where ϕ_b is the undrained shear strength parameter.

It is possible to re-derive the above factor of safety equations using the shear strength equation for an unsaturated soil. As the method of slices is used in the analysis, the mobilized shear force at the base of slice, S_m, can be written,

$$s_m = \frac{\beta}{F}\left(c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b\right) \qquad \text{..................(16)}$$

The normal force at the base of a slice, N is derived by summing forces in the vertical direction:

$$N = \frac{W + (X_R - X_L) - \frac{c'\beta\sin\alpha}{F} + \frac{u_a\beta\sin\alpha\,(\tan\phi - \tan\phi^b)}{F} + \frac{u_w\beta\sin\alpha\tan\phi^b}{F}}{\cos\alpha + \frac{\sin\alpha\tan\phi}{F}} \qquad \text{..................(17)}$$

when only moment equilibrium is satisfied, the factor of safety equation can be written as,

$$F_m - \frac{\sum\left(c'\beta R + \left[N - u_w\,\beta\frac{\tan\phi^b}{\tan\phi'} - u_a\,\beta\left(1 - \frac{\tan\phi^b}{\tan\phi'}\right)\right] R\tan\phi'\right)}{\sum W_x - \sum Nf} \qquad \text{..................(18)}$$

The factor of safety equation with respect to horizontal force equilibrium can be written as,

$$F_f = \frac{\sum\left(c'\beta\cos\alpha + \left[N - u_w\,\beta\frac{\tan\phi^b}{\tan\phi'} - u_a\,\beta\left(1 - \frac{\tan\phi^b}{\tan\phi'}\right)\right]\tan\phi'\cos\alpha\right)}{\sum N\sin\alpha} \qquad \text{..................(19)}$$

when the soil is saturated, the ϕ^b term must be set equal to ϕ'.

3.3 Quake Analysis: The governing motion equation for dynamic response of a system in finite element formulation can be expressed as:[9]

$$[M]\{\ddot{a}\} + [D]\{\dot{a}\} + [K]\{a\} = \{F\} \qquad \text{..................(20)}$$

where [M] mass matrix; [D] damping matrix; [K] stiffness matrix; $\{\ddot{a}\}$ vector of nodal acceleration; $\{\dot{a}\}$vector of nodal velocities; $\{a\}$vector of nodal displacement.
The vector of loads could made up by different forces:

$$\{F\} = \{F_b\} + \{F_s\} + \{F_n\} + \{F_g\} \qquad \text{..................(21)}$$

where: $\{F\}$ is vector of load; $\{F_b\}$ body force; $\{F_s\}$ force due to surface boundary pressure; $\{F_n\}$ concentrated nodal force; and $\{F_g\}$ earthquake loads.
The mass matrix can be a consistent mass matrix or lumped mass matrix. The consistent mass matrix:

$$[M] = \int_v \rho < N >^T <N>\,dv \qquad \text{..................(22)}$$

The lumped mass matrix:

$$[M] = \int_{v} \rho \, [\psi] dv \qquad \text{.................(23)}$$

where ρ mass density; <N> row vector of interpolation functions; {ψ} a diagonal matrix of mass distribution factors.
Damping matrix assume to be a linear combination of mass matrix and stiffness matrix:

$$[D] = \alpha \, [M] + \beta \, [K] \qquad \text{.................(24)}$$

where α & β are scalars and called Rayleigh damping coefficients. They can be related to a damping ratio η by:

$$\eta = \frac{\alpha + \beta \omega^2}{2\omega} \qquad \text{.................(25)}$$

where ω is the particular frequency of vibration for the system.
The Reyleigh damping coefficient was taken from QUAKE/W program, by using the lowest and the second lowest system frequencies and a constant damping ratio. These values is supposed to represent the dynamic situation in the Mosul Dam area.
The stiffness matrix is:

$$[K] = \int_{v} [B][C][B] dv \qquad \text{.................(26)}$$

where [B] is strain displacement matrix; [C] constitutive matrix.

3. CASE STUDY

In this study application of numerical solution for saturated-unsaturated flow of water and stability analysis of MOSUL dam. Finite element method based on unsaturated soil theory and slope stability analysis based on concept of slices. Three conditions of water rapid drawdown were applied which represent the critical cases stated in the objectives (section 3, introduction).

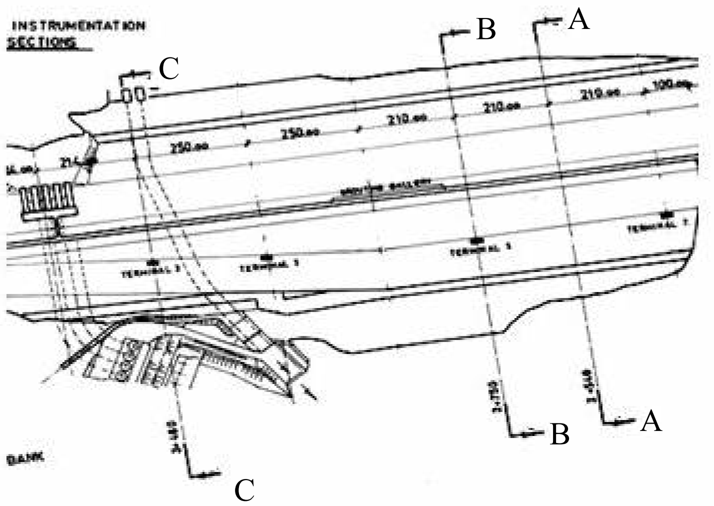


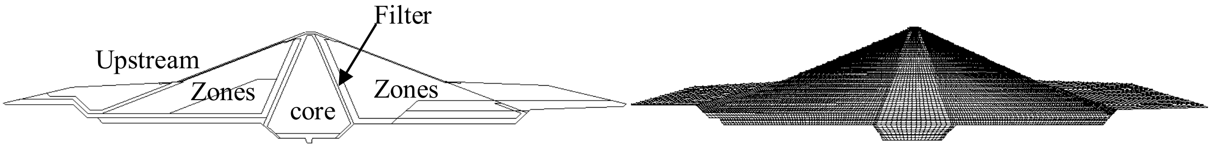
Figure (1): Plane of MOSUL Dam

MOSUL dam is an earth dam, 3.6 Km long, maximum high of more than 113 m, dam width of 650 m and crest level is 343 meters above sea level, located about 50 Km north of MOSUL city on Tigris river. Figures (1) and (2) show the plan and cross-sections of the dam respectively.

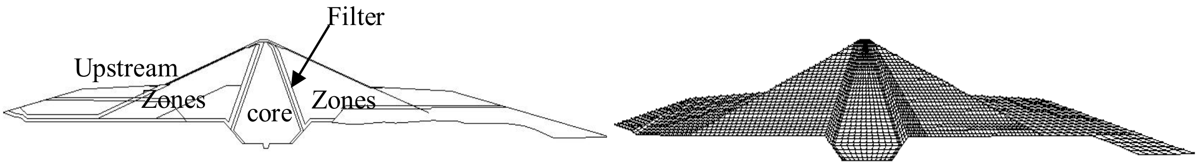
4.1 Seepage Analysis

The dame site in this study is modeled by SEEP/W model, which could be used to analyzed both simple and complex seepage problems. SEEP/W model uses the finite element method for two dimensional darcy's flow in both saturated-unsaturated soils. The major differences between water flow in saturated and unsaturated soil are (i) the coefficient of permeability is not a constant but is a

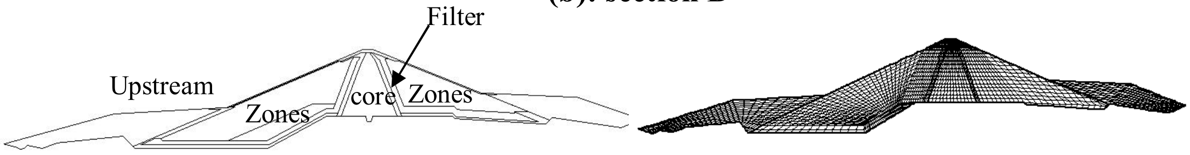
function of degree of saturation or matric suction in unsaturated soils, and (ii) the volumetric water content of unsaturated soil can be vary with time.[10]



(a): section A



(b): section B



(c): section C

Figure (2): Cross-Sections and Selected Element Mesh of MOSUL Dam

The application of dam model was made by sketching the earth dam cross section which is discreditized into a finite element mesh consisting of triangular and quadrilateral regions, using quadrilateral element as shown in Figure (2). Each element in the mesh must be associated with a soil type and boundary condition. The material Properties of the Shell, Filter, and Clay core are shown in Table (2).Water evacuation time during transient rapid drawdown is shown in Table (3).

Table (2):Shell materials, Filter, and Clay core, properties[average results obtained from the Mosul dam directory documents]

No.	Parameter	Unit	Shell	Filter	Core
1.	Dry density	kN/m^3	19.5	19.5	17.88
2.	Natural density	kN/m^3	20.5	20.5	21.3
3.	L.L	-	-	-	38
4.	Angle of internal friction	Deg.	37	37	29.5
5.	Cohesion (c)	kN/m^2	0	0	26
6.	Permeability	m/sec	$1.69*10^{-5}$	$1.69*10^{-3}$	$3.5*10^{-11}$
7.	Poisson ratio	-	0.25	0.25	0.35
8.	Modules of elasticity	kN/m^2	69000	69000	7616

Table (3): The water evacuation time during transient rapid drawdown at 8,21 and 30 days

Step #	Initial	1	2	3	4	5	6	7	8	Water

										evacuation Time
Time (day)	0	2	6	10	14	18	22	26	30	30
	0	2	6	10	14	18	22			21
	0	2	4	8						8

The grain size distributions, hydraulic conductivity functions, and soil water characteristic curve of the soils are shown in Figures (3),(4) and (5). Soil water characteristic curves of the zone and core material are predicted using Fredlund and Xing method[8]. The function of water drawdown at time (30, 21, 8) day are shown in Figure (6).

4.2 Slope Stability Analysis

Slope stability analysis, using computer modeling software SLOPE/W were conducted base on three methods: Bishop, Morgenstern-Price, and Lowe-Karafiath by dividing the soil mass above slip surface into vertical slices, with seepage analysis done using SEEP/W.

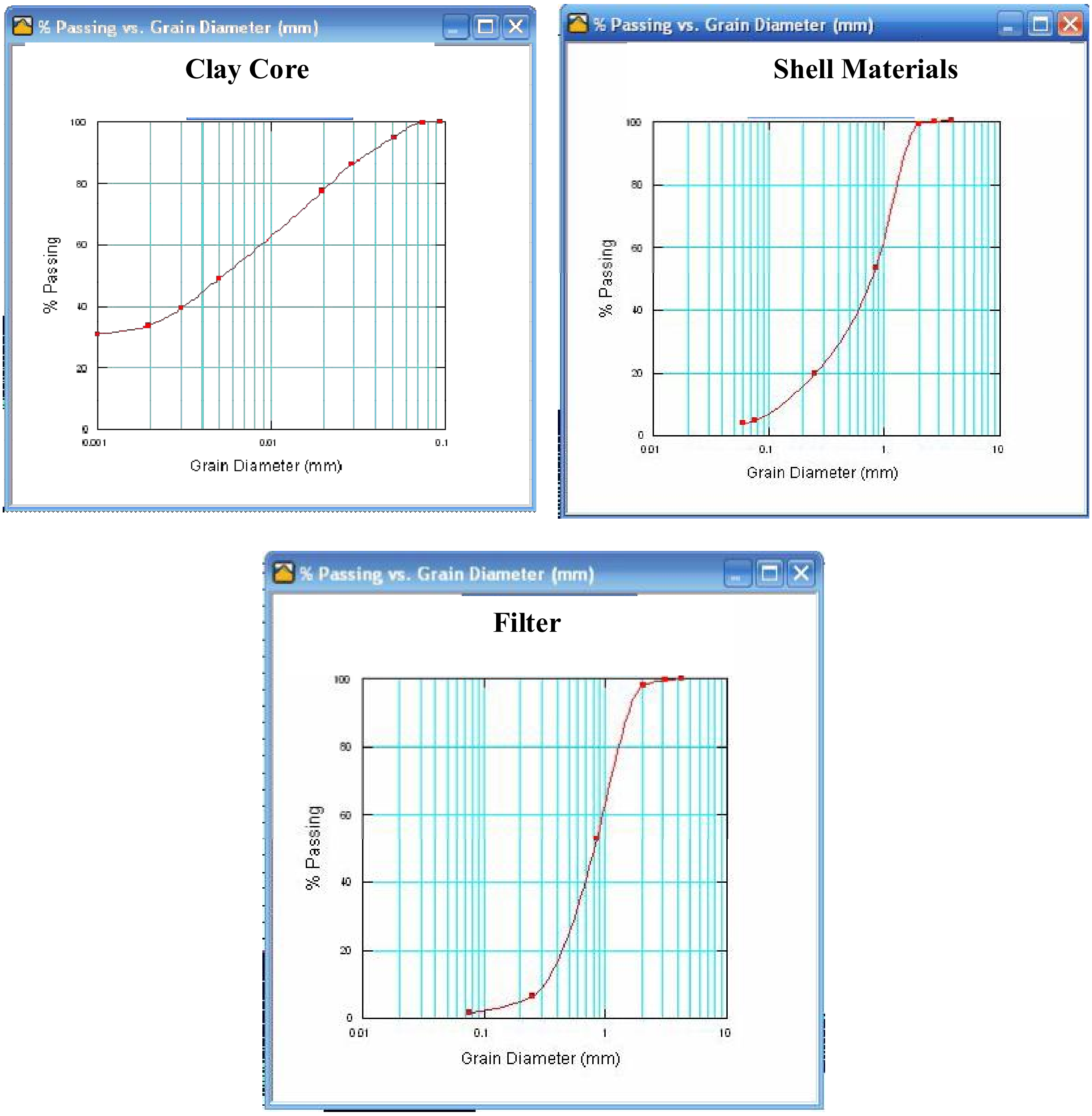


Figure (3): Grain Size Distribution of the Soils.

It should be noted that;

- Bishop method satisfies only moment equilibrium consider normal forces but not shear force between the slices.
- Morgenstern-Price method satisfies both force and moment equilibrium. The direction of the inter slices force is set equal to the average of the ground surface slope at the top of the slice and surface slop at the bottom of the slice.
- Lowe-Karafiath method satisfies only force equilibrium uses a selected inter slice force function f(x)=Half-sine.

The transient saturated-unsaturated seepage output model used as data input in the slope satiability analysis. Mohr-Coulomb failure criteria used for soil models to simulate the shear strength characteristic of a soil with three values of unsaturated shear strength parameter ϕ_b (0, 0.5ϕ, ϕ) .

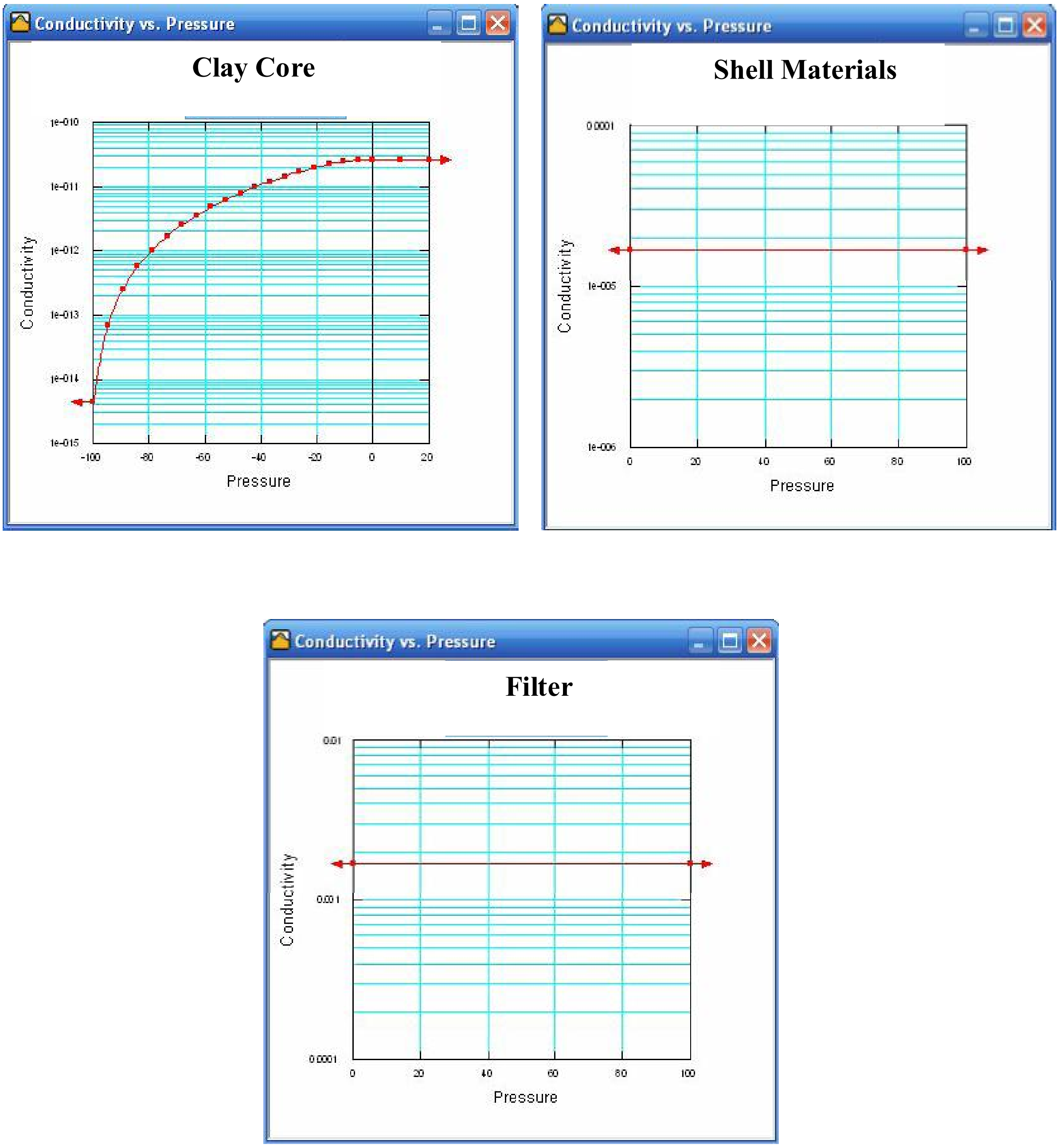


Figure (4): Conductivity Function For the Soils Used

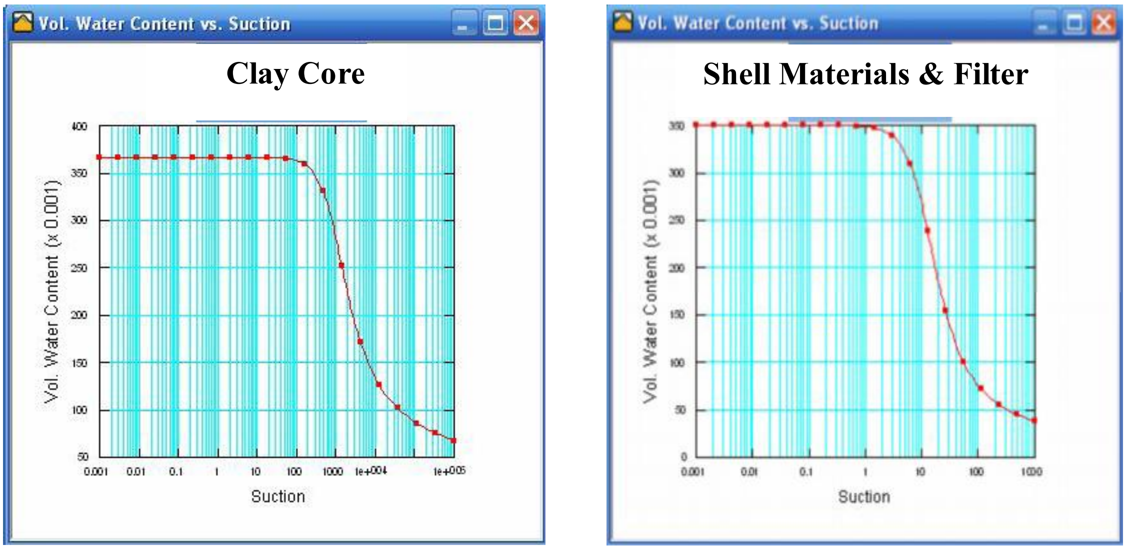


Figure.(5): Soil Water Characteristic Curve for the Soils Used

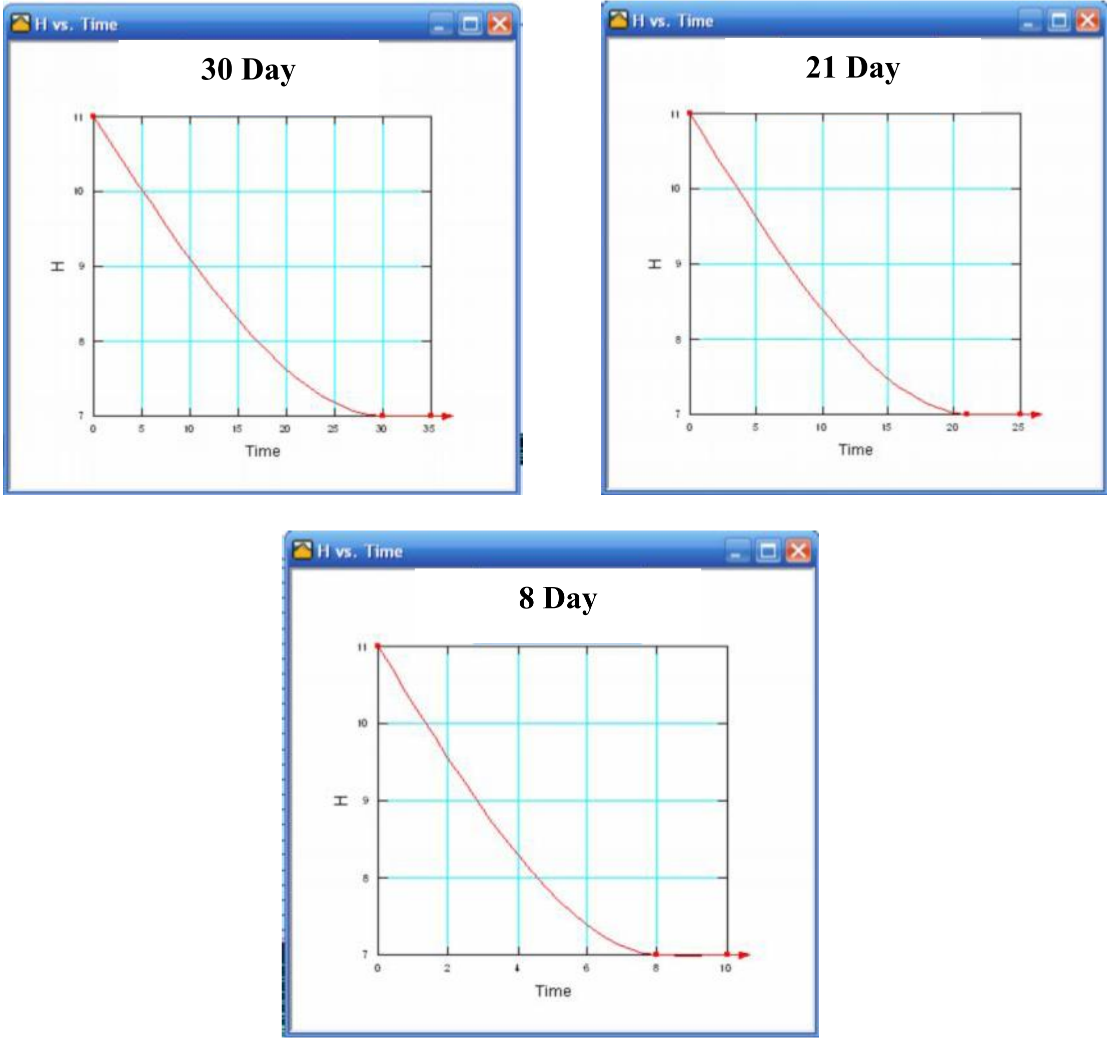


Figure (6): Function of Water Evacuation Time Drawdown With Time

4.3 Dynamic Stability Analysis

Dynamic analysis of MOSUL dam was performed using transient saturated-unsaturated seepage output model at the end of time of rapid drawdown water level as data input in the quake analysis. Pore pressure function Pore pressure function are defend as shown in Figure (7) which are used during earthquake shaking. Earthquake recorded imported from GEO-SLOPE library for 10 sec corresponds to an acceleration of 0.34g as shown in Figure (8). [9]

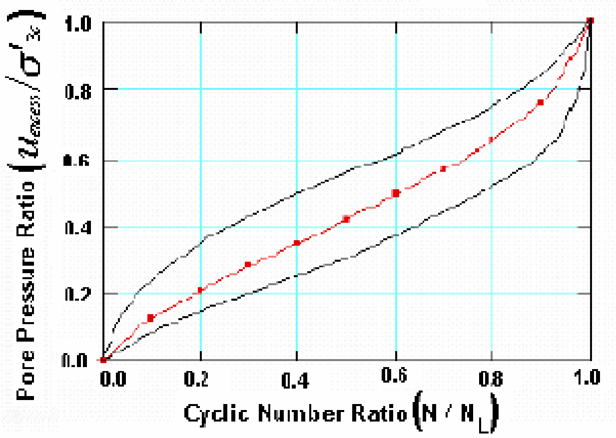


Figure (7): Pore Pressure Function [9]

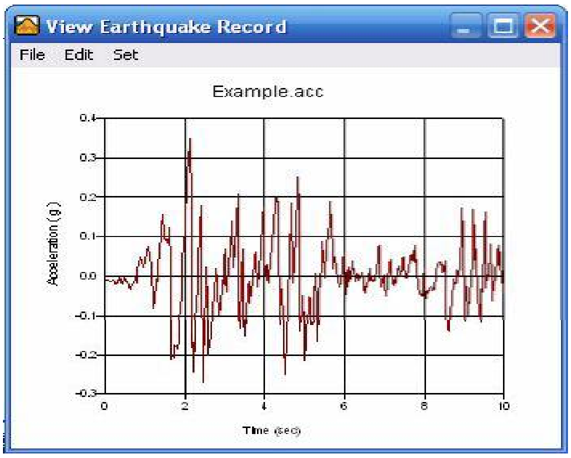


Figure (8): Earthquake Recorded [9]

Where N is the stress cycles; and N_L is the stress cycles required to produce liquefaction in Figure (7).

5. RESULTS OF THE ANALYSIS

Results of the slope stability analysis of the dam (factor of safety) considering the water drawdown level at time (8, 21 and 30) day are shown in Figure (9). The minimum slope stability factors of safety was achieved within the second day in the case of 8 day water drawdown. These values for the indicated case (8 days) were less than 2 (FOS< 2), while the factors of safety were ranging between 2-2.5 for the other cases (water evacuation time 21 and 30 days). Consequently, the case of (8) day drawdown is very critical condition compared with the two other drawdown cases. It is worth mentioning here that the values of the slope stability factors of safety of the cases (30 and 21 days of water drawdown) are nearly the same for the three sections (secA, secB, secC) applying the three indicated methods of calculation. Transient flow was also studied considering the steady state value as the initial condition, and then water level was assumed to decrease according to the selected drawdown steps shown in Table (2). In this case, the FOS values was found to be low in the first days and then, it increases at the end of the water evacuation time of 8,21 and 30 days. It is clear that the minimum FOS values were obtained considering a steady state flow case through saturated soils. Here, it could be concluded that the lowest values of FOS (FOS< 1.0) was obtained at the second day for drawdown case of 8 days which represent the most dangerous case that should be considered.

On the other hand, Figure (9) also show the values of slope stability factor of safety for three selected cross section (secA, secB, secC). These Figure indicate that the factors of safety are more critical in the dam parts near sections (A and B) and is more safe near sec (C) for selected analytical slope stability methods and selected unsaturated shear strength parameters (ϕ_b). This is obvious due to the height difference between these dam parts.

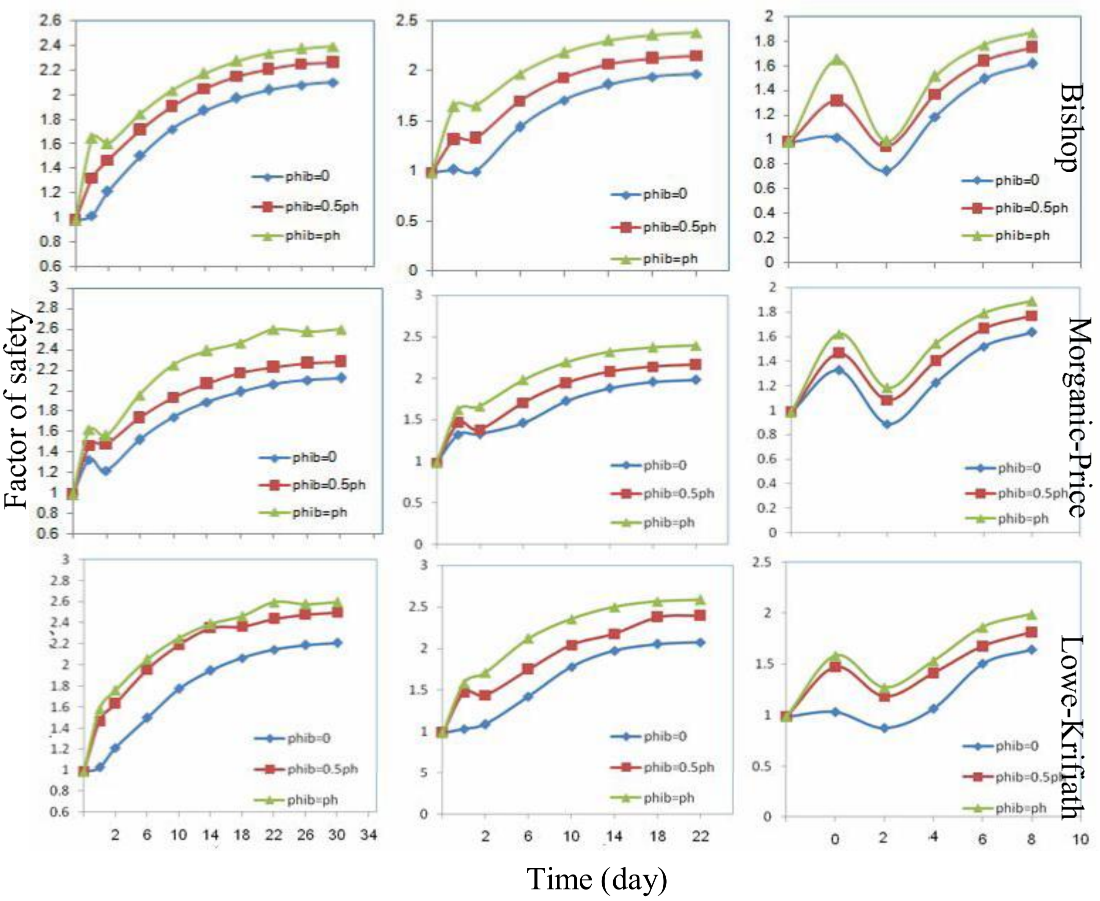


Figure 9 (a): Dam Section A

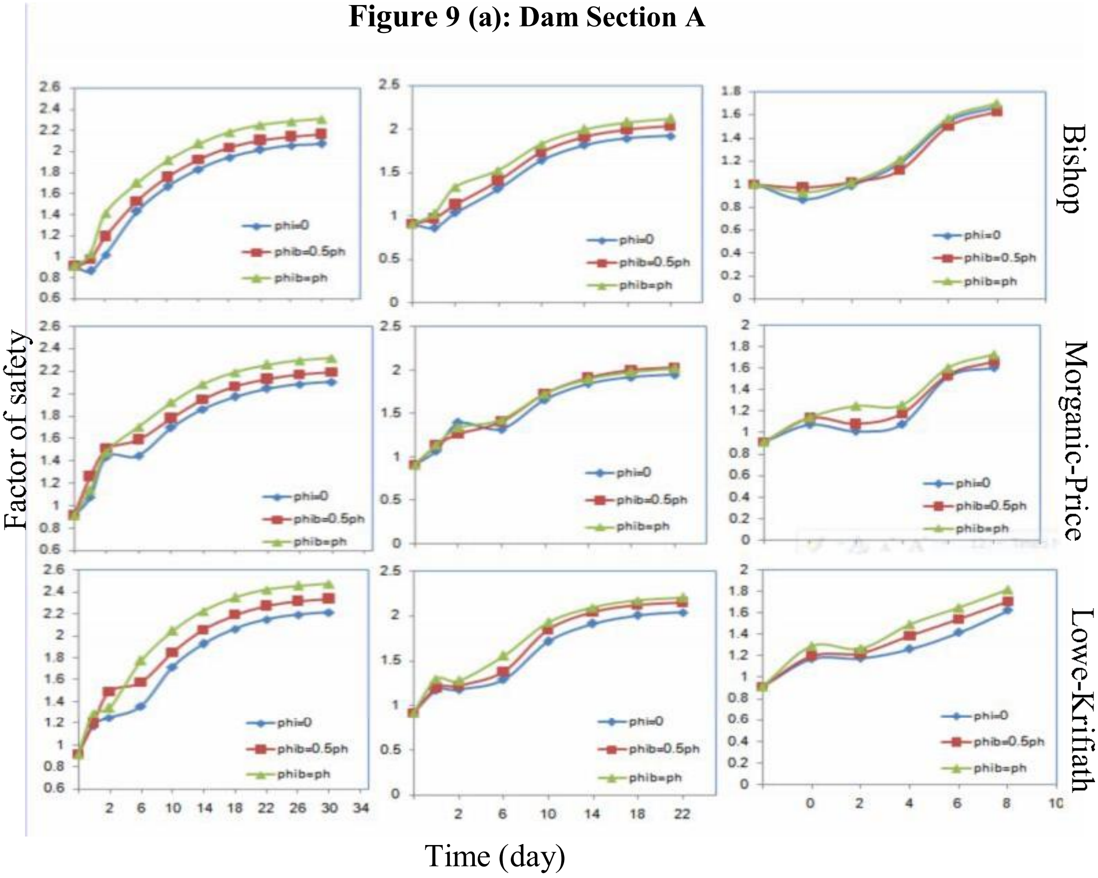


Figure 9 (b): Dam Section B

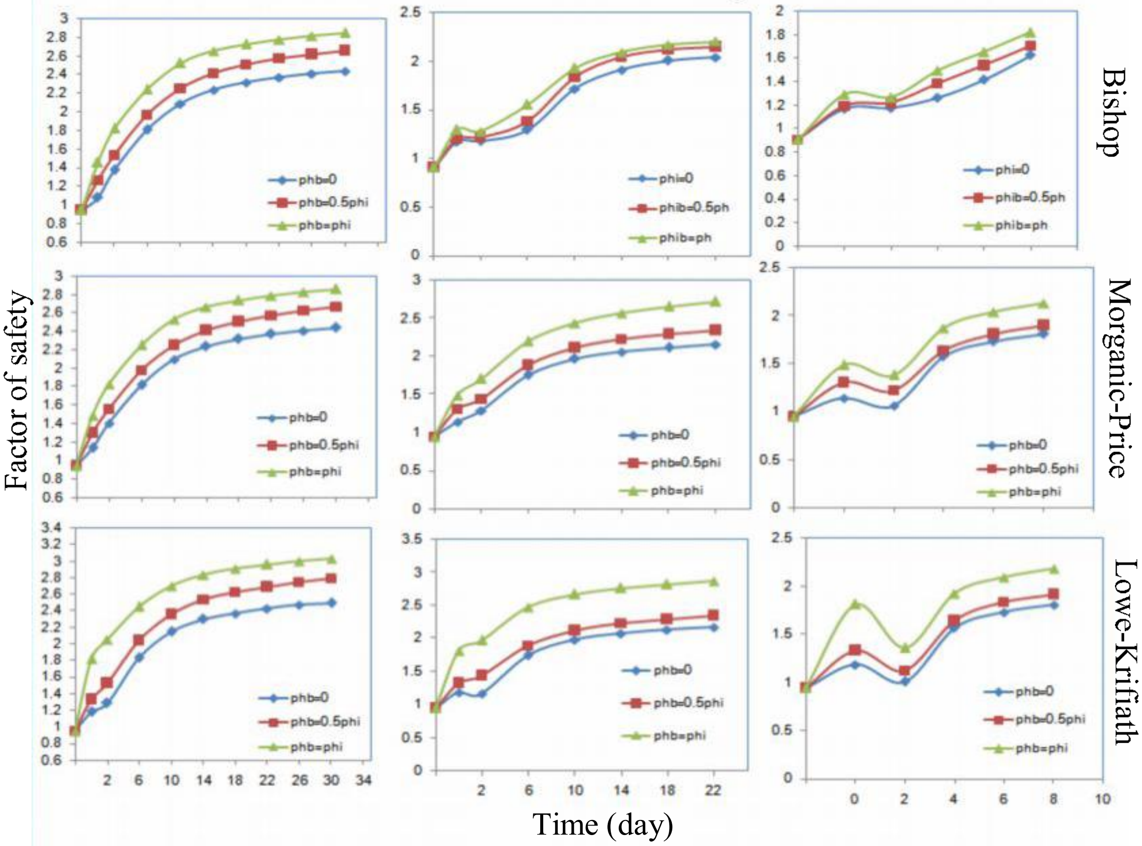


Figure 9 (c): Dam Section C

Figure (9): For dam sections (A,B & C) factor of Safety For the studied water drawdown cases (8,21 and 30) days

Table (3): Stability factors of safety of Mosul dam during the earthquake shaking at the end of 2nd day

	Sat.(steady)	Unsat.(steady).	30 day	21 day	8 day
	Section A				
Ph$_b$=0	0.912	0.963	0.858	0.772	0.527
Ph$_b$=0.5ph	1.017	1.003	0.98	0.86	0.683
Ph$_b$=ph	1.061	1.08	1.03	0.96	0.866
	Section B				
Ph$_b$=0	0.923	0.996	1.015	0.972	0.756
Ph$_b$=0.5ph	1.066	1.08	0.931	0.823	0.783
Ph$_b$=ph	1.074	1.146	0.983	0.883	0.758
	Section C				
Ph$_b$=0	0.986	0.985	0.911	0.869	0.724
Ph$_b$=0.5ph	1.097	1.095	1.043	1.066	1.086
Ph$_b$=ph	1.166	1.150	1.074	1.100	1.18

Concerning the method of slope stability analysis, Figure (9) also show that the highest slope stability factors of safety were reached using Morgenstern-Price method. This is due to the consideration of both force and moment equilibrium in the analysis, while the two other methods either force and moment equilibrium in the analysis. while in the two other methods either force and moment equilibrium is considered in the analysis.

The effect of unsaturated shear strength parameters used for the slope stability factor of safety was also shown in the same Figure indicated that increasing the values of (ϕ_b) leads to a more safe conditions against sliding with the most critical condition in the saturated case.

Finally, the response of Mosul dam during the earthquake shaking was obtained at the second day for drawdown cases (30, 21, 8 day) which represent the most critical and dangerous case, in term of slope stability factors of safety presented in Table (3). This table shows a very critical factors of safety, i.e FOS near and below 1.0, for all studied dam sections and for all drawdown times and (ϕ_b) values.

6. CONCLUSIONS

The obtained results indicated a critical situation of the dam from slope stability point of view during 8 day water drawdown and within the second day especially near sections A and B of the saturated earth dam . The effect of considering the unsaturated conditions was found to increase the FOS.

The response of Mosul dam during the earthquake shaking at the end of the time of the rapid drawdown water levels at (30, 21, 8) day in term of slope stability factors of safety is presented in Figure. Stability factors of safety of Mosul dam during the earthquake shaking at the end of 2nd day are presented in Table (3). This table shows a very critical factors of safety, i.e FOS near and below 1.0, for all studied dam sections and (ϕ_b) values.

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