ANALYTICAL STUDY OF REINFORCED CONCRETE TWO-WAY SLABS WITH AND WITHOUT OPENING HAVING DIFFERENT BOUNDARY CONDITIONS

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Abstract

Concrete slabs with opening are usually designed with help of traditional rules of thumb proposed by building codes. Such methods, however, introduce limitations concerning size of openings and magnitude of applied loads. Furthermore, there is a lack of sufficient information and instructions are needed to design opening in slabs of different boundary conditions in existing concrete slabs. The aim of this research is to carry out finite element analyses by using the ANSYS 5.4 program with a non-linear concrete model satisfying complex support condition to predict the ultimate load for the different types of reinforced concrete slabs. The effects of openings for different types of boundary conditions were studied and show that the opening in slabs which having supported on four edges have little effects on slab. Boundary conditions also studied here which show the slabs fixed on two opposite edges at least have clearly behavior on slab compared with another boundary conditions. Opening also have a great effect on values and distribution of normal stresses in slabs especially at opening region.

دراسة تحليلية للبلاطات الخرسانية المسلحة باتجاهين مع وبدون فتحات لشروط حدود مختلفة

محتبه محتبه حسام عبدالله دحام مدرس مساعد قسم الهندسة المدنية كلية الهندسة حكريت

الخلاصة

السقوف الخرسانية ذات الفتحات تصمم بالاعتماد على قوانين وطرق تقليدية معدة من قبل مدونات البناء. تلك الطرق تقدم محددات عن حجم الفتحات ومقدار الحمل المسلط. كما أن هنالك نقص في المعلومات المطلوبة لتصميم السقوف ذات الفتحات لشروط حدود مختلفة. الهدف الرئيسي لهذا البحث هو لتطبيق نظرية العناصر المحددة باستخدام برنامج ANSYS مع استخدام نموذج الخرسانة اللاخطي على نماذج ذات شروط حدود معقدة لتتبع قيمة الحمل الأقصى لأنواع مختلفة من السقوف الخرسانية المسلحة. تأثير الفتحات لشروط حدود مختلفة قد تم دراستها, وأظهرت الدراسة بأن وجود الفتحات في سقوف مسندة من الجهات الأربع لها تأثير قليل على السقوف. كما تم دراسة تأثير شروط الحدود على السقوف وبينت الدراسة أن تصرف السقوف المسندة بثبات كلي من جهتين متقابلتين على الأقل يكون أكثر وضوحا مقارنة مع شروط حدود أخرى. الفتحة أيضا لها تأثير كبير على قيم وتوزيع الاجهادات العمودية في السقوف خاصة عند منطقة وجود الفتحة.

Key words: Two-Way Slab, Finite Element, Reinforced Concrete, Opening

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Notation							
<i>u</i> , <i>v</i> , <i>w</i>	The displacement components.		{d}	Nodal displacement vector.			
[B]	Strain displacement matrix.		$\{d\}_e$	Column vector of nodal displacements.			
[D]	Constitutive law matrix.		<i>{f}</i>	Nodal force vector			
$[K]_e$	Element stiffness matrix		$\{\varepsilon\}_e$	Column vector of nodal strains.			
[L]	Differential operator matrix		{e}	Nodal strain vector.			
[N]	Shape function matrix		$\{\sigma\}_e$	Axial stress vector.			
$\{d^*\}_e$	Column vector of virtual nodal		$\{U\}_e$	Displacement vector at any point within			
	displacements			the element.			

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Introduction:

Slabs, in definition, are structures that transmit load normal to their plane. Concrete slabs are widely inuse as floors not only in industrial and residential buildings but also as decks in bridges.

Two way slabs are a form of construction unique to reinforced concrete, among the major structural materials. It is an efficient, economical, and widely used structural system. It is supported on all four sides and the length is less than twice the width. The slab will deflect in two directions, and the loads on the slab are transferred to all supports.

Although there are several different variations of two-way slabs, they can be generally described as one or a combination of three two-way systems: flat plates, flat slabs, and two-way beam-supported slabs. The selection of the most advantageous location for a floor opening depends on the type of two-way slab.

Behavior of A Slab Loaded to Failure in Flexure

There are four or more stages in the behavior of a slab loaded to failure: (see figure 1). 1-Before cracking, the slab acts as an elastic plate and, for short-time loads the deformations, stresses, and strains can be predicted from an elastic analysis.

2- After cracking and before yielding of the reinforcement, the slab is no longer of constant

stiffness, since the cracked regions have a lower flexural stiffness, EI, than the uncracked regions; and the slab is no longer isotropic since the crack pattern may differ in the two directions, Although this violates assumptions in the elastic theory still predicts the moments adequately. Normal building slabs are generally partially cracked at service loads.

3-Yielding of the reinforcement eventually starts in one or more regions of high moment and spreads through the slab as moments are redistributed from yielded regions to areas that are still elastic.

4-Although the yield lines divide the plate to form a plastic mechanism, the hinges jam with increased deflection and the slab forms a very flat compression arch ,as shown in figure (2), this assumes that the surrounding structure is stiff enough to provide reactions for the arch [1].

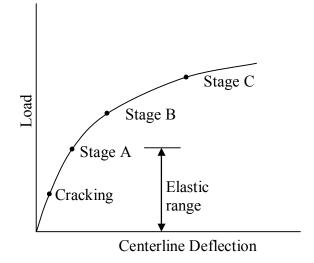


Figure (1) Load-Deflection Diagram^[1].

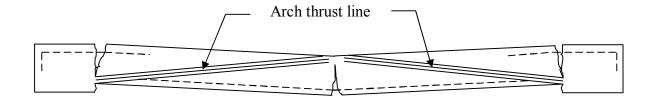


Figure (2) Arch Action in Slab^[1].

Opening in New Slabs

Opening in slabs are usually required for plumbing, fire protection pipes, heat and air conditioning. Larger openings that could amount to the elimination of a large area within a slab panel are sometimes required for stairs and elevators shafts. For newly constructed slabs, the locations and sizes of the required openings are usually predetermined in the early stages of design and accommodated accordingly.

Section 13.4.1 of ACI 318-08^[2] permits openings of any size in any new slab system, provided that an analysis is performed that demonstrates that both strength and serviceability requirements are satisfied. The analysis for slabs containing openings could be complex and time consuming, as an alternative the ACI 318-08^[2] code gives guidelines and limitations for opening location and size. If the designer satisfies those requirements the analysis could be waived.

Modifications to an existing structure, although not frequent, occur in almost every structure. New slab openings or penetrations in an existing concrete building are easily accommodated in the majority of instances. However, the analysis required, and the remedies are typically more involved than similar openings in a new slab.

Relative Previous Studies

In 1961, Wood^[3] was considered as the leader among researchers to develop a rigorous mathematical solution for the analysis of membrane action in clamped and simply supported reinforced concrete circular slabs. The slabs were considered to be isotropically reinforced in the radial and circumferential directions and subjected to uniformly distributed loading. The analysis was based on establishing a yield criterion containing the membrane stresses. The relationship between the bending moment (M) and the membrane force (N), taken to act at mid-depth, was given in non-dimensional form:

$$\frac{M}{M_o} = 1 + \alpha \left(\frac{N}{T_o}\right) - \beta \left(\frac{N}{T_o}\right)^2$$

where:

$$\left(\alpha = \frac{\frac{1}{2} \cdot \frac{h}{d} - \frac{3}{2} \cdot t}{1 - \frac{3}{4} \cdot t}\right), \left(\beta = \frac{\frac{3}{4} \cdot t}{1 - \frac{3}{4} \cdot t}\right) \text{ and } \left(t = \frac{A_s}{d} \cdot \frac{f_y}{f_c'}\right)$$

Wood also conducted some tests on reinforced concrete square slabs of (1.727m) length and (57.1 mm) thickness, restrained at the boundaries and subjected to 16 point loading distributed over the entire surface of the slab to represent the case of uniform loading.

In 1971, Hung and Nawy^[4] presented a method to solve the problem of membrane action in uniformly loaded isotropically reinforced concrete rectangular slabs having various boundary conditions by adopting the conventional yield line pattern for the collapse mechanism and making use of the work method. The theoretical predictions were compared with results of tests conducted on twenty-nine slab models and the theoretical to experimental load ratio varied from (1.11) to (0.4) with a mean value of (0.76).

In 1978, Al-Hassani^[5] outlined new concepts for the plastic behavior of materials with tension cracks based on flow rules and applied them to problems of axially restrained concrete slabs. He presented a theoretical as well as experimental investigation on the elastic-plastic behavior of R.C. slab strips including the effect of the elastic shortening of the strips, the elastic deformation of the surrounding elements in addition to the effect of physical gap at the supports.

In 1990, Vecchio^[6] and Collins investigated the ultimate load carrying capacity of an orthotropically reinforced concrete flat slab subjected to uniform loading. The structure was analyzed by using a computer program taking into account material non-linearity (for both concrete and reinforcement), geometric non-linearity, membrane action, temperature degradation of material strength and various other influencing factors. Results indicate that non-linear effects in reinforced concrete slabs and most notably membrane action can result in affect floor load capacities more than the design values.

In 2005, Al-shimmary^[7] studied the effect of membrane action in uniformly loaded, isotropically reinforced concrete rectangular slabs either fixed along two parallel edges with one edge simply supported and one edge free, or simply supported along two parallel edges with one edge fixed and one edge free. In either type, two cases were investigated depending on whether the free edge of the slab is a short or a long edge; the slabs were found to carry loads more than the corresponding loads predicted by Johansen's simple yield line theory.

In 2006, Salman^[8] studied the effect of membrane action in uniformly loaded orthotropically reinforced concrete rectangular slabs having three fixed edges with one simply supported edge, two cases were investigated depending on whether the simply supported edge of the slab is short or long edge. The slabs were found to sustain loads more than those predicted by Johansen's yield line theory .

In the same year, Sahagian^[9] studied the effect of membrane action in uniformly loaded, isotropically reinforced concrete rectangular slabs either fixed along two adjacent edges with one edge simply supported and one edge free; or simply supported along two adjacent edges with one edge fixed and one edge free. In either type, two cases were investigated depending on whether the free edge of the slab is short or a long edge; study showed that the slabs can sustain loads higher than those predicted by Johansen's yield line theory.

In the same year, Yaseen^[10] studied the effect of membrane action in uniformly loaded orthotropically reinforced concrete rectangular slabs restrained along two adjacent edges and simply supported along the other two edges. The slabs were found to carry loads higher than those predicted by Johansen's simple yield line theory.

In 2006, Abd AlRazaak^[11] studied the effect of membrane action in orthotropically reinforced concrete rectangular slabs restrained on two opposite sides and simply supported along the remaining sides, subjected to a uniformly distributed load. The slabs were found to

carry loads higher than those predicted by Johansen's simple yield line theory and the ratio of yield load to yield line theory collapse load was greatest for thin slabs.

Finite Element Analysis of Slabs

The finite element method essentially approximates slab behavior by subdividing the plate continuum into a mesh of discrete finite elements. Plate or shell elements are typically employed to represent the behavior of slabs by deformations at the mid-surface. Figure (3) shows the shear forces $(q_x \text{ and } q_y)$ and bending $(m_x \text{ and } m_y)$ and twisting (m_{xy}) moments resulting from transverse load q for an infinitesimal plate element [12]. Note:-

⊗ and ● Indicate Shear Forces Into The Plane and Out of The Plane ,Respectively) [12].

Also figure (4) shows a typical triangular plate element with three degrees-offreedom at each node (ω_N is the out - of-plane translation and $\partial \omega_{\scriptscriptstyle N}/\partial x$ and $\partial \omega_{\scriptscriptstyle N}/\partial y$ are the two rotations about the y- and x- axes, respectively. at the Nth node) corresponding element nodal forces. Nodal displacements for a plate element are acquired by solving the global structure equilibrium equation. Element nodal displacements can then be used to compute internal forces needed for slab design, usually based on one of two approaches: moment fields using moment curvature relations (the classical approach) or element nodal forces using the element stiffness matrix^[12].

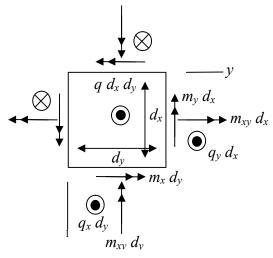
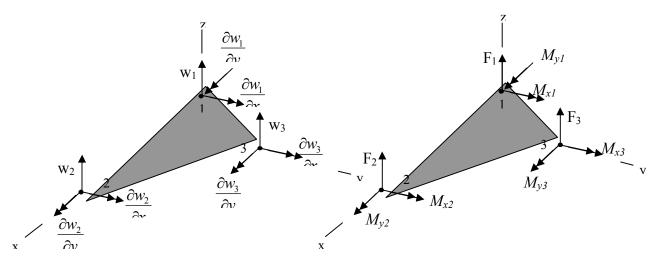


Figure (3) An Infinitesimal Plate Element Shown With Resulting Shear Forces and Moments Due to Transverse Loading



(a) Three Degrees of Freedom at Each Node

(b) Element Nodal Forces

Figure (4) A Typical Triangular Plate Element Used to Model Slabs^[12].

In ANSYS 5.4 terminology, the term model generation usually takes on the narrower meaning of generating the nodes and elements that represent the spatial volume and connectivity of the actual system. Thus, model generation in this discussion will mean the process of defining the geometric configuration of the model's nodes and elements^[13].

Basic Finite Element Relationships

The basic steps is the derivation of the element stiffness matrix, which relate the nodal displacement vector, $\{d\}$, to the nodal force vector, $\{f\}$.

Considering a body subjected to a set of external forces, the displacement vector at any point within the element, $\{U\}_e$ is given by:

$$\{U\}_e = [N].\{d\}_e$$
(1)

where, [N] is the matrix of shape functions, $\{d\}_e$ the column vector of nodal displacements. The strain at any point can be determined by differentiating equ. (1):

$$\{\varepsilon\}_e = [L]. \{U\}_e$$
(2)

where, [L] is the matrix of differential operator. In expanded form, the strain vector can be expressed as:

$$\{\varepsilon\} = \begin{cases} \varepsilon_{x} \\ \varepsilon_{y} \\ \varepsilon_{z} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{cases} = \begin{cases} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial w}{\partial z} \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\ \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \end{cases}$$
.....(3)

substituting equ (1) into (2) gives:

$$\{\varepsilon\}_e = [B]. \{d\}_e$$
(4)

where, [B] is strain-nodal displacements matrix given by:

$$[B] = [L].[N]$$
(5)

the stress vector can be determined by using the appropriate stress-strain relationship as: $\{\sigma\} = [D] \{e\}$

 $\{\sigma\}_{e}=[D].\ \{\varepsilon\}_{e}$ (6)

where, [D] is the constitutive matrix and $\{\sigma\}_e$ is:

$$\{\sigma\}_e = \begin{bmatrix} \sigma_x & \sigma_y & \sigma_z & \tau_{xy} & \tau_{yz} & \tau_{zx} \end{bmatrix}^T \qquad \dots (7)$$

From equ (4) and equ. (6), the stress-nodal displacement relationship can be expressed as:

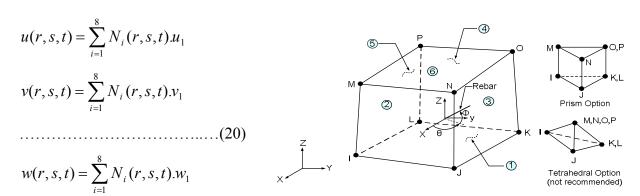
$$\{\sigma\}_e = [D]. [B]. \{d\}_e$$
 (8)

For writing the force-displacement relationship, the principal of virtual displacements are used. If any arbitrary virtual nodal displacement, $\{d^*\}_e$, is imposed, the external work, W_{ext} , will be equal to the internal work W_{int} .:

Solid 65 Element Description

In ANSYS 5.4 program, SOLID65 (or 3-D reinforced concrete solid) is used for the 3-D modeling of solids with or without reinforcing bars (rebar). The solid is capable of cracking in tension and crushing in compression. In concrete applications, for example, the capability of the solid element may be used to model the concrete, while the rebar capability is available for modeling reinforcement behavior. The element is defined by eight nodes having three degrees of freedom at each node: translations of the nodes in x, y, and z-directions. Up to three different rebar specifications may be defined.

The most important aspect of this element is the treatment of nonlinear material properties. The concrete is capable of cracking (in three orthogonal directions), crushing, plastic deformation, and creep. This 8-node brick element is used, in this study, to simulate the behavior of concrete layer. The element is defined by eight nodes and by the isotropic material properties. The geometry, node locations, and the coordinate system for this element are shown in Figure (5)^[13]. For this element, the displacements field are represented by:



where, ui, vi and wi are the displacement components of node i, and Ni (r, s, t) is the shape function at node i,

$$N_i(r, s, t) = \frac{1}{8}(1 + r.r1).(1 + s.s1).(1 + t.t1) , .(r.r1 + s.s1 + t.t1 - 2)(21)$$

where, $r_i = \pm 1$, $s_i = \pm 1$, $t_i = \pm 1$

LINK 8 Element Description

LINK8 is a spar (or truss) element which may be used in ANSYS 5.4 program in a variety of engineering applications. This element can be used to model trusses, sagging cables, links, springs, etc. The 3-D spar element is a uniaxial tension-compression element with three degrees of freedom at each node: translations of the nodes in x, y, and z-directions. As in a pin-jointed structure, no bending of the element is considered. Plasticity, creep, swelling, stress stiffening, and large deflection capabilities are included. This element used to simulate the behavior of reinforcement bars and thus it is capable of transmitting axial force only. The geometry, node locations, and the coordinate system for this element are shown in Figure (6)^[13].

Also, Solid65 element can be used to analyze problems with reinforced bars. Up to three rebar specifications may be defined. The rebar's are capable of plastic deformation and creep. The rebar orientation is defined by two angels measured with respect to the element's coordinate system. See Figure (5).

Experimental Verification

To ascertain the validity of the used element, slab with opening was analyzed. This slab was tested by others and sufficient experimental data is available for their proper modeling by the finite element method.



Figure (6) LINK8 Geometry [13]

Experimental Slabs

Piotr [15] tested a series of two-way concrete slabs with different dimensions of opening and different methods of strengthening. The tested slabs are subjected to uniformly distributed loads . Here we will analyze the square two-way slab of dimension $2.6 \times 2.6 \times 0.1$ m which have an opening of dimension $1.2 \times 1.2 \times 0.1$ m as shown in figure (7). It is assumed in

calculations that the slab is supported in the corners and elastically in four intermediate points along the edge. The intermediate supports are modeled as non-linear springs.

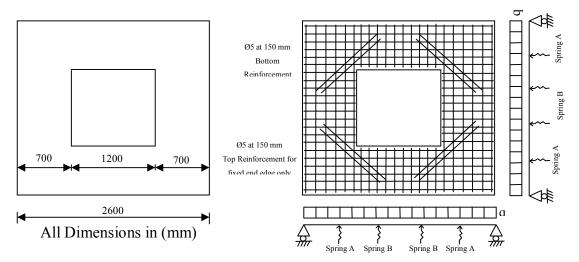


Figure (7) Tested^[15] Two-Way Reinforced Concrete Slab With Opening

Table (1) gives the material properties used in the analysis of the slab, and table (2) gives the properties for the two types of spring support (A and B) used in the analysis.

Table (1) Material Properties Used in The Analysis of The slabs.

Material	Property	Slabs
Reinforcing Steel	Yield Stress (MPa)	510
	Young's Modulus (MPa)	209×10^{3}
	Poisson's Ratio	0.3
	Top Reinforcement for fixed end edge only	Ø5 at 150mm
	Bottom Reinforcement	Ø5 at 150mm
Concrete	Compressive Cube Strength (MPa)	45.4
	Tensile Strength (MPa)	
	Young's Modulus (MPa)	3.0
	Poisson's Ratio	34×10^{3}
		0.2

Table (2) Spring Properties, Non-linearity of The Springs Represents Improvement of The Supporting Conditions.

Displacement-force data for spring A			
Displacement (m)	Force kN		
0	0		
0.0018	2.2		
0.0044	12		
0.0065	20		

Displacement-force data for spring B			
Displacement (m)	Force kN		
0	0		
0.0018	3.6		
0.0032	12		
0.0045	20		

Results and Discussion

Results of the analysis are shown in Figure (9) represent the load mid-span deflection curve of slab. As can be seen from Figure, a reasonable comparison between the computed and experimental values. The results show good agreement between the computed and experimental values. Figure (10) shows no. of elements through depth of slab.

No.4

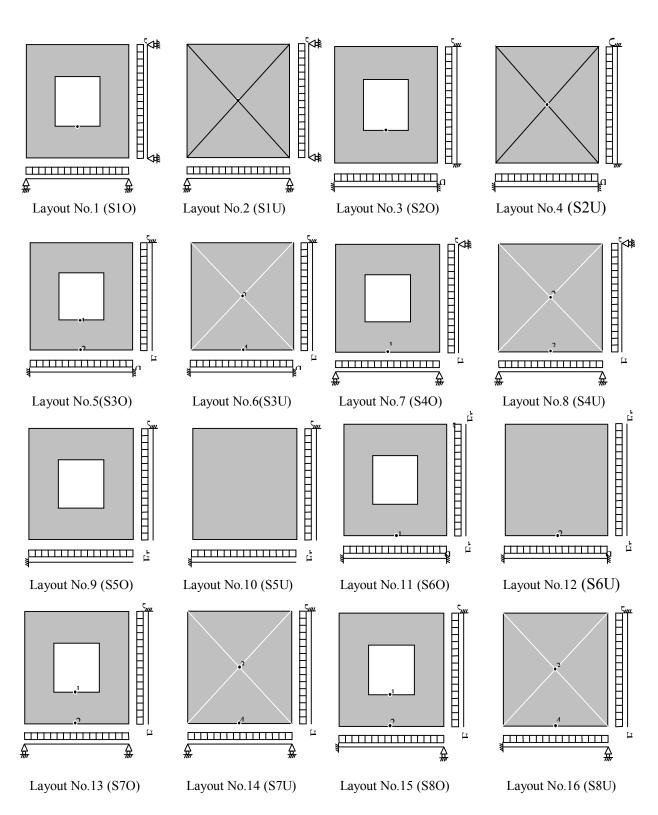
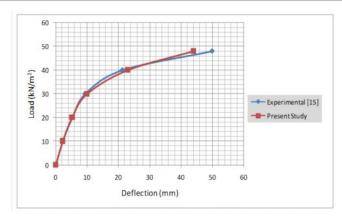


Figure (8) Slabs Layout



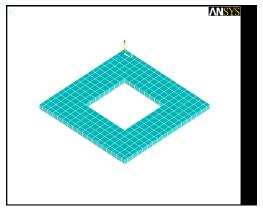


Figure (9) Load Deflection Curves

Figure (10) Finite Element Mesh

Effect of Boundary Conditions:

Effect of supports status on the deflection of the slabs with and without opening is studied here. The results in Figures (13),(14),(15),(16),(23) and (24) shows that the deflection in slab behave linearly with fixed supports on opposite two edges at least, while Figures (17) and (18), show the curve begin with linear stage which represent about 21.43% from the total load with small deflection (about 2.9-4.3%) from the maximum deflection then the slope of line change.

From the Figure, (11), (12), (19), (20),(25),(26),(27), and (28), three distinct stages can be defined for the slabs of simply supported on two opposite edges at least. First stages, the slabs behaves linearly which represent about (11.4-28.5)% from the total load with small deflection (2.2-2.8)% from the maximum deflection. Second stage represents the part of the curve that the slabs behaviour nonlinear, this stage represent about (17.1-28.2)% from the total applied load with deflection represent about (15.5-22.5)% from the maximum deflection. In the third stages, it can be seen that the curve becomes linearly, this stage represent about (43.3-71.5)% from total applied load.

Effect of Openings

Openings effects also studied here for slabs observing that the surface area of slab without opening is larger than slab with opening. It can be seen from Figures (11), and (13) the maximum deflection for the slab with opening which supported by fixed edges from all sides represent about 5.26% from the deflection of slab supported by simply supported from all sides at ultimate pressure, this percentage approximately equal to the percentage of slabs without opening (about 5.4%) with same boundary conditions as shown in Figures (12) and (14).

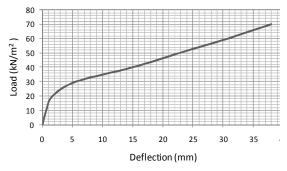


Figure (11) Load Deflection Curves For Slab S1O

Figure (12) Load Deflection Curves For Slab S1U.

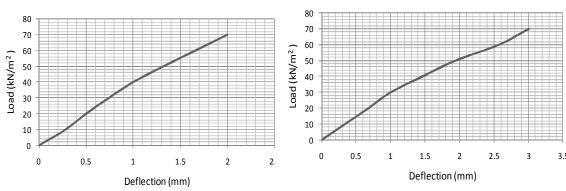


Figure (13) Load Deflection Curves For Slab S2O

Figure (14) Load Deflection Curves For Slab S2U.

From Figures (16), and (19) it can be seen the maximum deflection for the slab with opening which fixed supported at two opposite edges and fixed-free at the other two opposite edges represent about 9.6% from the deflection of slab supported by simply supported at two opposite edges and simply supported-free at the other two opposite edges at ultimate load, while this percentage equal to 35% for slabs without opening for the same boundary conditions as shown in figures (18) and (20).

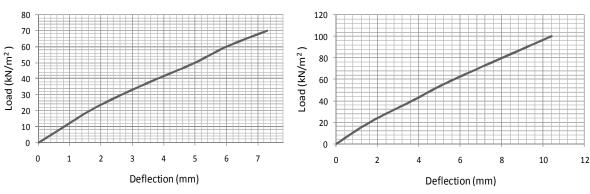


Figure (15) Load Deflection Curves For Slab S3O at Point 1

Figure (16) Load Deflection Curves For Slab S3O at Point 2

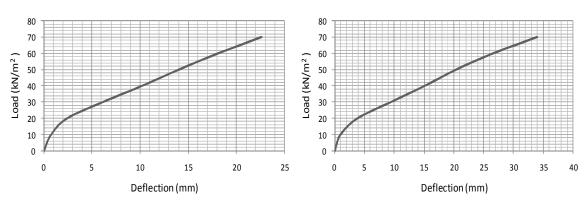
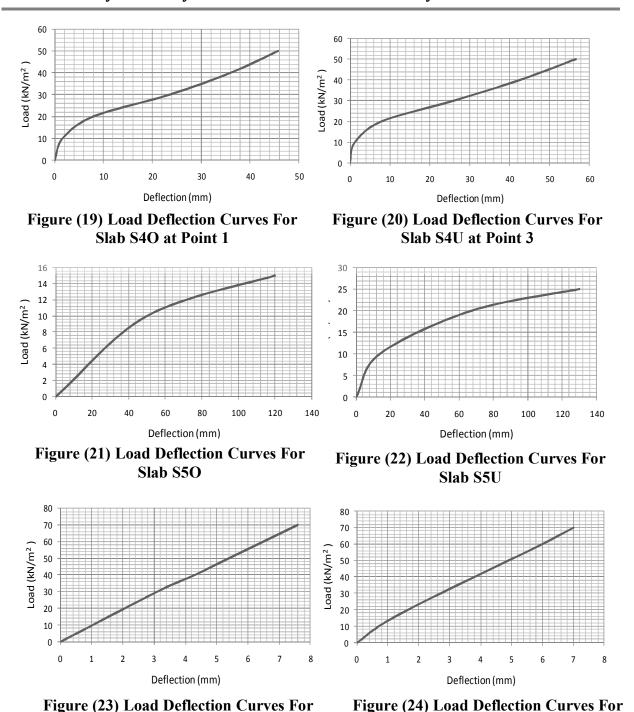


Figure (17) Load Deflection Curves For Slab S3U at Point 3

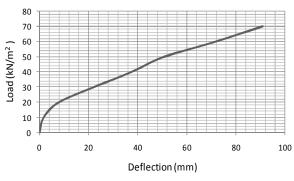
Figure (18) Load Deflection Curves For Slab S3U at Point 4



Figures (26), and (30) show the maximum deflection for the slab with opening which supported by simply supported-fixed at two opposite edges and fixed-free at the other two opposite edges represent about 48.75% from the deflection of slab supported by simply supported at two opposite edges and fixed-free at the other two opposite edges at ultimate pressure, while this percentage equal to 35.2% for slabs without opening for the same boundary conditions as shown in Figures (28) and (32).

Slab S6U at Point 2

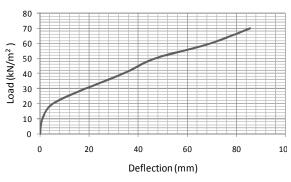
Slab S6O at Point 1



80 70 60 40 40 40 20 10 0 0 20 40 60 80 100 Deflection (mm)

Figure (25) Load Deflection Curves For Slab S7O at Point 1

Figure (26) Load Deflection Curves For Slab S7O at Point 2



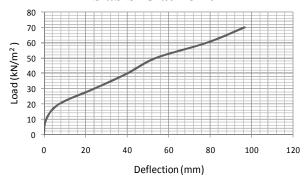
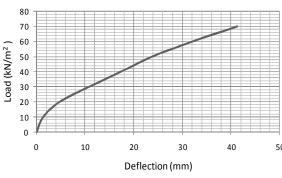


Figure (27) Load Deflection Curves For Slab S7U at Point 3

Figure (28) Load Deflection Curves For Slab S7U at Point 4



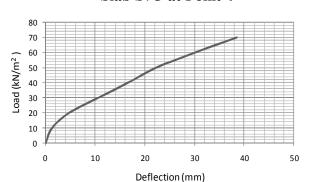
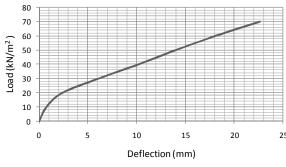


Figure (29) Load Deflection Curves For Slab S8O at Point 1

Figure (30) Load Deflection Curves For Slab S8O at Point 2



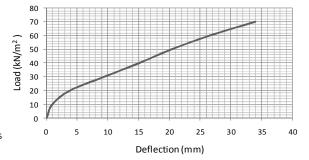


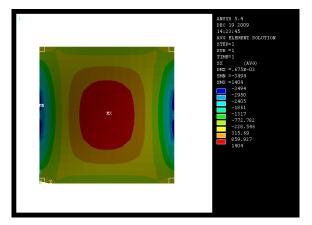
Figure (31) Load Deflection Curves For Slab S8U at Point 3 Stresses in Concrete

Figure (32) Load Deflection Curves For Slab S8U at Point 4

Normal stresses at the bottom face at load equal to $20~\rm{kN/m^2}$ were calculated throughout the slabs.

Normal stresses for slabs without opening having different boundary conditions (S2U, S3U, S4U) have nearly same behaviour, and Figure (33) shows the tension stresses spread along large area in mid zone of the slab. While for slabs without opening which having two adjacent edges fixed and the other two edges are free (S5U), and slabs without opening which having fixed supported at two opposite sides and free at other sides (S6U), have nearly same behaviour, and Figures (34), (35) show the tension stresses spread throughout all bottom surface slabs.

Figure (36) shows normal stresses for slab having an opening, it can be seen from figure that the tension stresses spread around opening, this behavior are same for all slabs having an opening.



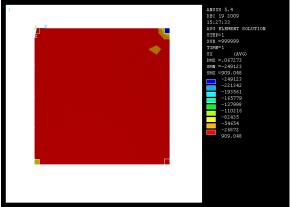
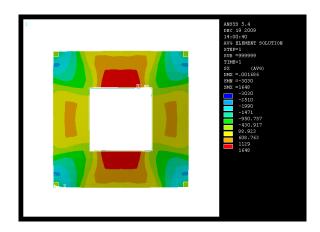
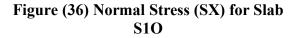


Figure (33) Normal Stress (SX) for Slab S2U

Figure (34) Normal Stress (SX) for Slab S5U





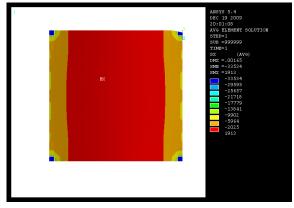


Figure (36) Normal Stress (SX) for Slab S6U

Conclusions and Recommendations

The finite element method using ANSYS 5.4 program which is capable of modelling the behaviour of the reinforced concrete two-way slabs. The program yields good results as demonstrated by the analysis of slabs. It is evident from figures that opening in the slab reduced its strength, where, as expected, the presence of the opening reduced the stiffness of the slab.

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The behaviour of load-deflection curves for the slabs with and without opening having fixed supported on two opposite edges at least, more clearly from the other curves have another boundary conditions.

An increase of stresses values at opening edges can be noted clearly, and supported status also have a great effect on values and distribution of these stresses.

Further studies will be needed to verify the behaviour of two-way slabs under different loading conditions, also effect of dynamic loading and the effect of opening shape. The moment distribution and the strengthening needs around openings could be analyzed for different types of slabs.

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