

## Stream flow Simulation and Synthetic Flow Calculation by Modified Thomas Fiering Model

Dr. Cheleng A. Arselan  
College Of Engineering  
University of Kirkuk

### Abstract

In this paper , Thomas-Fiering T-F model was used twice for simulation of Khassa Chi river in Kirkuk city by using historical monthly stream flow sequences for a period (1941-2001) . A modification to Thomas-Fiering model was done by extracting the persistency from the monthly flow values of Khassa Chi river and including the regression values between the monthly values of the flow without persistence. It was concluded that Thomas-Fiering model is very suitable to simulate Khassa Chi behavior. The modified model MT-F was more capable to reverse the monthly and annual statistical parameters especially the monthly and annual standard deviation and it was more sensitive in reversing the drought times which reflects its capability in other simulation operations for variables which are suffering from drought times.

Key words :Thomas-Fiering , Persistence Simulation

### نمذجة تصارييف الانهار وتخمين تصارييفها باستخدام النموذج الرياضي المحور ثوماس فيرينك

د.جلنك اكرم عمر أرسلان  
جامعة كركوك/كلية الهندسة

### الخلاصة

في هذا البحث استخدم النموذج ثوماس فيرينك مرتين لنمذجة نهر خاصة جاي الجاري في مدينة كركوك حيث تم اجراء تحليل لخصائص السلسلة الشهرية المسجلة لنهر خاصة جاي والمتضمنة تصارييف النهر الشهرية للفترة (1941-2001) . تم تحديث النموذج وذلك باستخلاص الخاصية الغير العشوائية من التصارييف الشهرية ويجاد معاملات الارتباط الخاصة بين التصارييف الشهرية الخالية من هذه الخاصية . تم التوصل الى قابلية النموذج الرياضي Thomas- Fiering بصيغته المعروفة لنمذجة نهر خاصة جاي كما انه عند تحسين النموذج الرياضي تم التوصل الى معاملات احصائية شهرية وسنوية وخصوصا قيم الانحراف المعياري الشهرية والسنوية اكثر قربا من البيانات

الاصلية. حيث بدا اكثر تحسسا لفترات الجفاف مما يجعله قابلا للاستخدام لعمليات النمذجة لمتغيرات اخرى مختلفة قد تعاني من فترات قحط.

الكلمات الدالة : ثوماس فيرينك الخاصية الغير العشوائية , النمذجة

## Introduction:

The design of water resources projects is based primarily on hydrologic and economic data .Stream flow records are major types of hydrologic data .Surface stream flow data have two major uses. The first is to provide general regional information .This data represents "natural" conditions and may be used in combination with similar data at other sites to gain general description of the stream flow of an area. The second major use is for project operation and purposes [7]. Hydrological data such as stream flow are ,however, essentially historical; that is observed in the past .The value of the record is directly related to its length .The aim in the collection of data is generally to determine the form of the relationship of dependent variable with an independent variable ,for instance the dependence of stream flow on time. Often the relationship is not fixed but contains an element of variation which can be described by a probability distribution .Having obtained the probability distribution of the observed events, the hydrologist is able to make an estimate of future events and from this attempt to solve his particular problem. One type of problem is that of estimating a future value of some stream flow characteristic such as peak flood or low flow.[4]Thomas and Fiering were the first to develop a mathematical model for sequential generation of stream flows. Their model treats the flow in any period as a linear function of the flow in the preceding period. This model is flexible, in that it can be used for weekly, monthly, or seasonal flows, as well as annual flow. It does not require that the flow data be normally distributed, and may be used with skewed distributions [5]. Thomas Fiering model was used by W.C.Boughton and Mckerchar, in 1968, for generating a synthetic stream flow records, using a data for 20 years from some New Zealand catchments [9]. Richard N.DeVeries, 1970 used Thomas Fiering model with addition to two modified similar models by the Army corps of Engineers Hydrologic Engineering Center, for simulation of Elkhorn river at Waterloo [7]. S.A.Robinson and F.G.Rohde, 1976 used many mathematical models to simulate and model the monthly stream flow in Federal republic of Germany for the Rhine river, Treena, Nires, Kizig rivers, the Thomas Fiering model was one of these models for this extensive study[8]. Receb Bakis, Ilke Gurkan 2004 converted the Thomas Fiering model into dimensionless form before using it in generating the synthetic flow series for Porsuk Cay in north of Turkey [6]. Ahmet kurunc,et al 2005 tested the performance of two approaches for forecasting water quality and stream flow data for Yisil Irmak river, in Turkey, one of these approaches was the Thomas Fiering method [1]. Ahmet Kurnc,Engin Yurtseven, 2005 used this model for estimating stream flow and water quality variables for Euphrates river in Turkey, the record data were taken from two stations and for a period (1984-1999)[2]. The present study was concerned with a new attempt in simulation and calculation of synthetic Khassa Chi flow sequences by using two approaches in applying the Thomas Fiering Model.

## Applied Methodology:

### A) Thomas- Fiering Model

In order to simulate Khassa Chi river records Thomas- Fiering approach was used in this study. Thomas Fiering Model presents a set of 12 regression equations .A well known Thomas Fiering model equation can be given as :

$$Y_{i+1} = \bar{Y}_{j+1} + r_{j,j+1} \frac{S_{j+1}}{S_j} (Y_i - \bar{Y}_j) + U_i S_{j+1} \sqrt{1 - r_{j,j+1}^2} \dots\dots\dots(1).$$

Where:

$Y_{i+1}$  is the value to be simulated for i+1 month from ith month.  $Y_i$  is the last observed value for the month i..  $\bar{Y}_{j+1}$  and  $\bar{Y}_j$  are mean monthly values during the j+1 and jth month

respectively.  $r_{j,j+1}$  is serial correlation coefficient between values in  $j$ th and  $j+1$  month.  $S_j, S_{j+1}$  are the standard deviations of monthly values during  $j, j+1$  months respectively.  $U_i$  is a random normal deviate with zero mean and unit variance [1]. As was mentioned before Thomas and Fiering use a linear regression model between flows in successive times. For example of using monthly flows, the flow in February is related to flow in January by a linear regression. Flow in March is related to the flow in February by another linear regression. Twelve linear regressions are used to describe the serial correlations of flows through the year, the final regression relating flow in January to flow in preceding December. The regression line relating flow in February to flow in January is given by:

$$Q_F - \bar{Q}_F = b(Q_j - \bar{Q}_j) \dots\dots\dots(2).$$

Where:

- $Q_F$ =Flow in February  $m^3/sec$ .
- $\bar{Q}_F$ =Mean of February flow  $m^3/sec$ .
- $Q_j$ =flow in January  $m^3/sec$ .
- $\bar{Q}_j$ =Mean of January flow  $m^3/sec$ .
- $b$ =regression line slope.

The regression line is inadequate by itself to describe the serial correlation between monthly flows. For example, if the starting flow in January was equal to the mean January flow  $\bar{Q}_j$ , this would generate the mean flow for February. This in turn would generate the mean March flow, and so on, with mean December flow generating the mean January flow and repeating the same sequence each year. Some random component is required in generating procedure to allow for the natural variations in flow which are not accounted for by the regression line. It should be noted that the February flows form a set of data with mean value  $\bar{Q}_F$  and variance  $S_F^2$ . Part of the variance is accounted for by regression line. If the correlation coefficient between February flows and January flows is  $r$ , then the proportion of the total variance of February flows that can be attributed to variation in January flows is equal to  $r^2$ .

$$\bar{S}_F^2 = \bar{S}_F^2 \cdot r^2 + \bar{S}_F^2 \cdot (1 - r^2) \dots\dots\dots(3).$$

Which means that?

Total variance =Variance explained by regression +Variance unaccounted for by regression. The standard error of estimate of the February flows from the January flows is  $S_f \cdot (1-r^2)$  and this is the measure of random or unexplained variation in February flows. Thomas and Fiering incorporated this additional variance into their generating model as follows :

$$(Q_F - \bar{Q}_F) = b_1(Q_j - \bar{Q}_j) + U \cdot S_F \cdot (1 - r^2)^{1/2} \dots\dots\dots(4).$$

Where  $U$  is a random number drawn from a standardized normal distribution with mean=0, and variance =1.

Forty-eight parameters are required to describe the 12 regression equations, which form the model:

- 12 values of mean monthly flow ( $\bar{Q}_j, \bar{Q}_F, \dots\dots\dots etc$ )
- 12 slopes of the regression lines ( $b_1, b_2, \dots\dots\dots etc$ )
- 12 correlation coefficients ( $r_1, r_2, \dots\dots\dots etc$ )
- 12 standard deviations ( $S_j, S_F, \dots\dots\dots etc$ )

A starting value is assumed for flow in the first month. A random number,  $U_i$  is selected and the value of flow in the second month is calculated from equation (4). A second random number is then used with flow in the next month to generate flow in the third month, and so on, Once started ,the generating procedure can be used to produce a synthetic record as long as is required.(W.C.Boughton,1968).

According to the above explanation of the regression between the months the Thomas – Fiering equation can be written again as :

$$Y_{i+1} = \bar{Y}_{j+1} + B_j(Y_i - \bar{Y}_j) + U_i S_{j+1} \sqrt{1 - r_{j,j+1}^2} \dots\dots\dots(5).$$

Where  $B_j$  is the regression coefficient for estimating the flow value in the j+1 month from the j month .[4].

**B) Modified Thomas Fiering Model.**

After a brief analysis to historical record of Khassa Chi river and determination of the river behavior , an attempt was done to transform the Thomas Fiering model by extracting the persistency from the river characteristics and by investigating the serial correlation between the random residuals of the monthly means of the historical record. The modified model can be explained as in the following equation:

$$RY_{i+1} = R\bar{Y}_{j+1} + RR_{j,j+1} \frac{RS_{j+1}}{RS_j} (RY_i - R\bar{Y}_j) + U_i RS_{j+1} \sqrt{(1 - RR_{j,j+1}^2)}. \dots\dots\dots(6).$$

Where:

$RY_i$  is the last randomized (without persistency) observed value for the month  $i$ ,  
 $RY_{i+1}$  is the randomized value to be simulated for  $i+1$  month from  $i$ th month.  
 $R\bar{Y}_j$  and  $R\bar{Y}_{j+1}$  are mean monthly values during the  $j$ th and  $j+1$  month respectively, also as randomized values.  $RR_{j,j+1}$  is serial correlation coefficient between randomized values in  $j$ th and  $j+1$  month.  $RS_j, RS_{j+1}$  are the standard deviations of the randomized monthly values during  $j, j+1$  months respectively.  $U_i$  is a random normal deviate with zero mean and unit variance .This equation can be written also as:

$$RY_{i+1} = R\bar{Y}_{j+1} + B_j (RY_i - R\bar{Y}_j) + U_i RS_{j+1} \sqrt{(1 - RR_{j,j+1}^2)}. \dots\dots\dots(7).$$

Where:

$B_j$  in this case is the regression coefficient for estimating the randomized flow value in the  $j+1$  month from the  $j$  month.

**Simulation of Khassa Chi River Record:**

It is necessary to understand the basic variables of stream flow before a model can be developed, for a given stream or river. Khassa Chi river is a tributary of Zghiton river, which is flowing into Adhaim dam reservoir. Analysis was done to a historical data of Khassa Chi river which was taken from hydrological study performed by Engineering consultancy bureau of Mustansiriyh University, college of Engineering at 2006[3]. The analysis on the historical data of the river record was done in order to understand the characteristics of the river .It was detected from the analysis that there are two components which are describing the river : persistence and random component .

**Persistence :**

Means a continuous effect after its cause is removed. It refers to the tendency for large flow events to follow large flows events, and for small flow events to follow the low flow

events. Another additional variance in stream flow from one month to the next that is not explained by persistence. This unexplained variance occurs at random and can be referred to as the random component. Any mathematical model for Khassa Chi River must include Persistence and random nature of the flow. Therefore the Thomas Fiering model was first applied in its general form eq.1 or eq.5 and then applied after transforming it by randomizing the monthly flow values by excluding the persistence values using eq.6 or eq.7.

**Generation and Calculation of Synthetic Khassa Chi river Record Requirements.**

A computer program has been written using MatLab programming tool for the calculation of synthetic record of monthly mean flow by the two mentioned models which are depending on the Thomas–Fiering model.

The following statistics of flow for each month were required :

- 1) Monthly mean values of flows with persistence component and without it.
- 2) Annual mean values of flows with persistence component and without it.
- 3) Monthly standard deviation values of flows with persistence component and without it.
- 4) Annual standard deviation values of flows with persistence component and without it.
- 5) Regression and serial correlation coefficients of flows with persistence component and without it.

Note : The serial correlation coefficient can be calculated as in the following equation .

$$RR_{j,j+1} = \frac{\sum_{i=1}^{i=N} (q_{i,j} - \bar{q}_j)(q_{i,j+1} - \bar{q}_{j+1})}{\sqrt{\left[ \sum_{i=1}^{i=N} (q_{i,j} - \bar{q}_j)^2 \right] \left[ \sum_{i=1}^{i=N} (q_{i,j+1} - \bar{q}_{j+1})^2 \right]}} \dots\dots\dots(8).$$

$q_{i,j}$  The j month flow of i year

$q_{i,j+1}$  The j +1 month flow of i year

$\bar{q}_j, \bar{q}_{j+1}$  : is the mean value of j's months flow and j+1's flows.

N: number of years of the historical record [6].

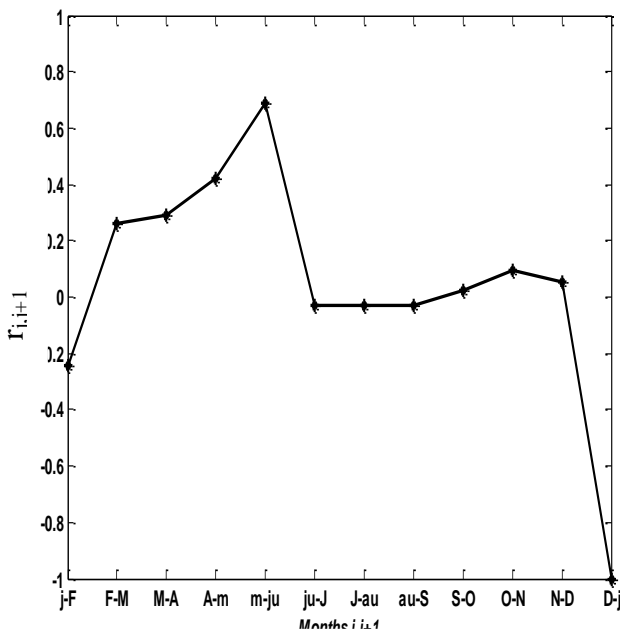
Note: In the present study the flow values here for the second approach will be for randomized flow values after the persistence extracting.

**A Simulation of Khassa Chi Reservoir Operation:**

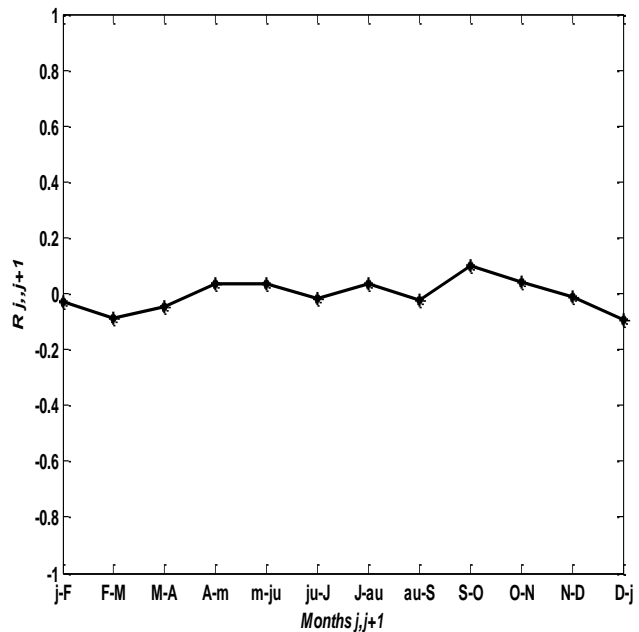
The determination of required capacity for Khassa Chi reservoir is usually called operation study. It is essentially a simulation of the reservoir operation for a period in accordance with an adopted set of rules. The operation study may analyze only a selected "critical period" of very low flow. An operation study may be performed with annual, monthly or daily time intervals. Monthly data are most commonly used, there for the historical data, and the generated one by Thomas–Fiering, which was applied in this study on Khassa Chi stream, will be suitable to perform this operation. Values of the cumulative sum of inflow minus withdrawals (including average monthly evaporation seepage) are calculated. The required storage for the interval is the difference between intial peak and the lowest through in the interval. The process was repeated for all cases in the period under study and the largest value of required storage is determined. The capacities of the Khassa Chi reservoir and the water levels for different demands were calculated and the results of this study will be shown in the next section.

**Results and Discussion:**

It is important to mention before the results reviewing that the historical record of Khassa Chi river was monthly record of flow for 61 years (1941-2001) this data was taken from (a report on Khassa Chi Dam project hydrological study by Dr.Ahmed N.Ali ,2005 [2]. The monthly Khassa Chi record for 56 years were used for an analysis purposes and the remained was used for the calibration of the simulation process. From the first application of Thomas Fiering Model to the Khassa Chi river. It was detected that the monthly Khassa Chi flow records are composed from two parts. Persistence and random part, this is very clear from the values of which were plotted in Figure(1) due to great values of  $r_{j,j+1}$  ,while Figure (2) represents the null persistency of the monthly flow values after the original part of the persistence was removed by non parametric method



Figure(1) Serial correlation coefficients between the monthly flows for Khassa Chi monthly flows with persistency part

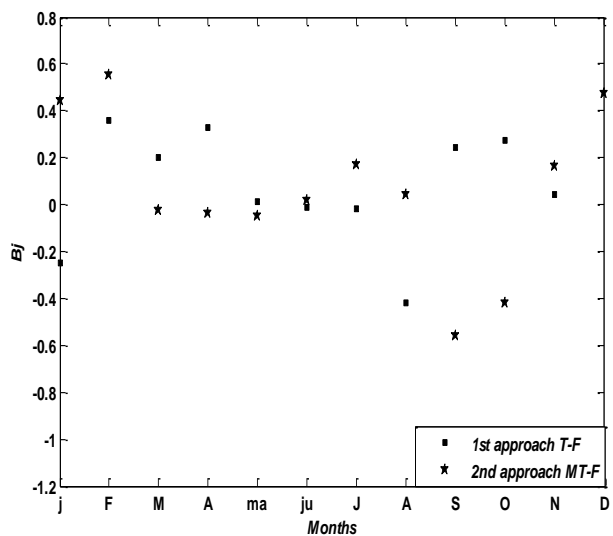


Figure(2) Serial correlation coefficients between the monthly flows for Khassa Chi monthly flows without persistency

The linear regression coefficients between sequenced monthly flows were used to relate each monthly flow value to the next monthly flow value , the regression values are found by depending on the historical flow values and shown in Figure(3) for the two approaches .

Note:The notation T-F means Thomas – Fiering model while MT-F means the modified Thomas-Fiering in the next figures and the table .

After using the required statistical values for simulating the Khassa Chi flow (monthly mean, monthly standard deviation), with addition to a generation process of normally distributed numbers for a required



Figure(3) The regression coefficients between the sequenced months for the two approaches

period, which were selected to be for monthly record of 65 years. The calculated information's above were used in a computer program written in Mat Lab to simulate and for synthetic flow calculation of the khassa Chi river, this was done separately to each application or approach of Thomas Fiering model. The synthetic monthly flows for 65 years of Khassa Chi river for the two approaches are shown in Figure (4) and Figure(5) respectively .

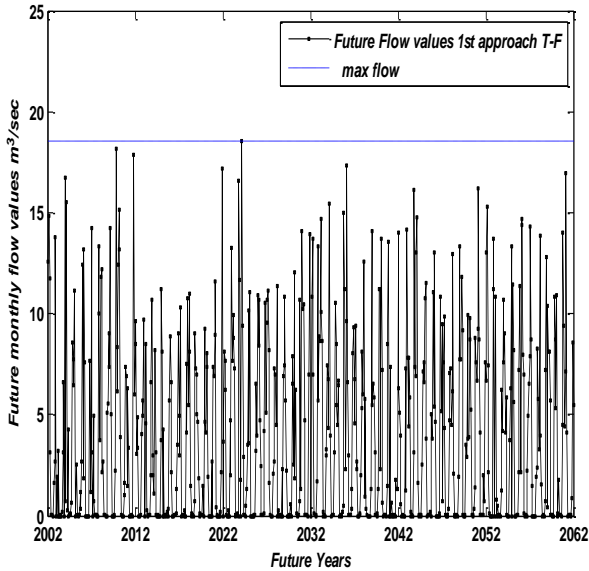


Figure (4) The Synthetic Khassa Chi flow ( the 1<sup>st</sup> T-F model)

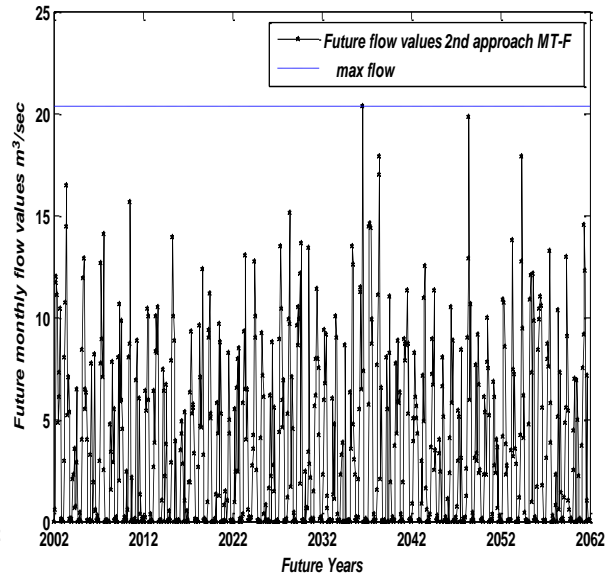


Figure (5) The Synthetic Khassa Chi flow (2<sup>nd</sup> MT-F model)

Figure(6,7)respectively show the monthly mean values and standard deviation comparison for the synthetic flows which are calculated by the two applied methods with the original data .

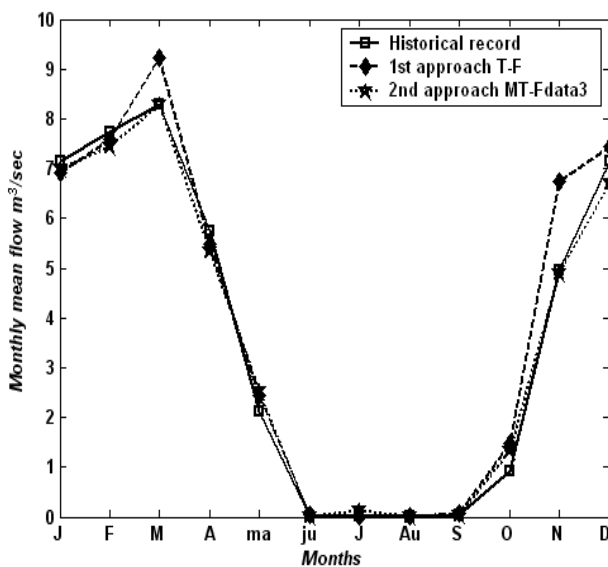


Figure (6) The comparison of Monthly mean flow of Khassa Chi for the synthetic data by two approaches

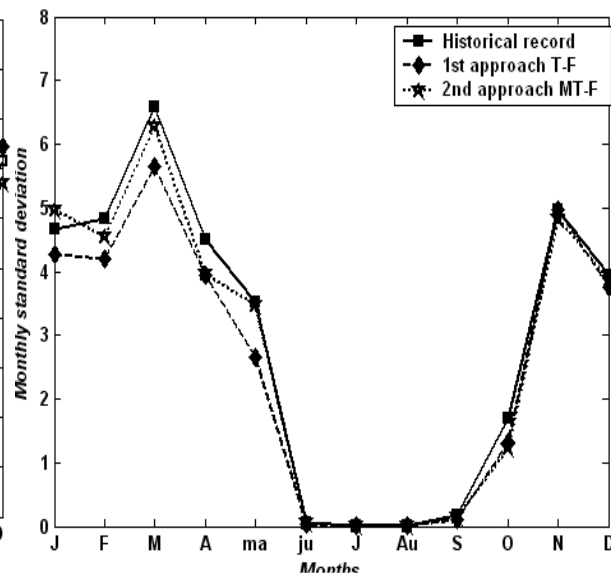


Figure (7) The comparison of Monthly standard deviation of Khassa Chi flow for the synthetic data by two approaches

Figure(8,9) show the comparison of the annual mean values and annual standard deviation values for the two applications also with the original data .

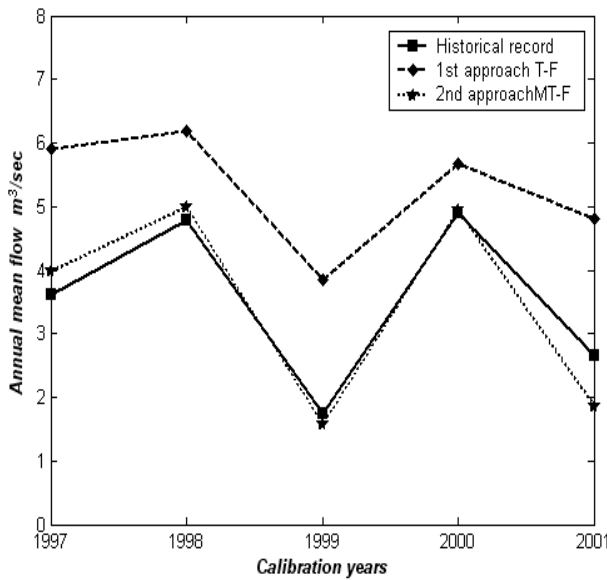


Figure (8) The comparison of Annual mean of Khassa Chi flow for the synthetic data by two approaches

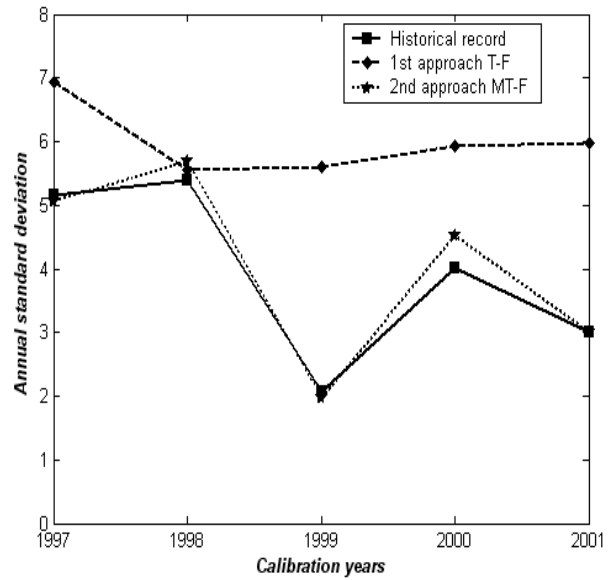
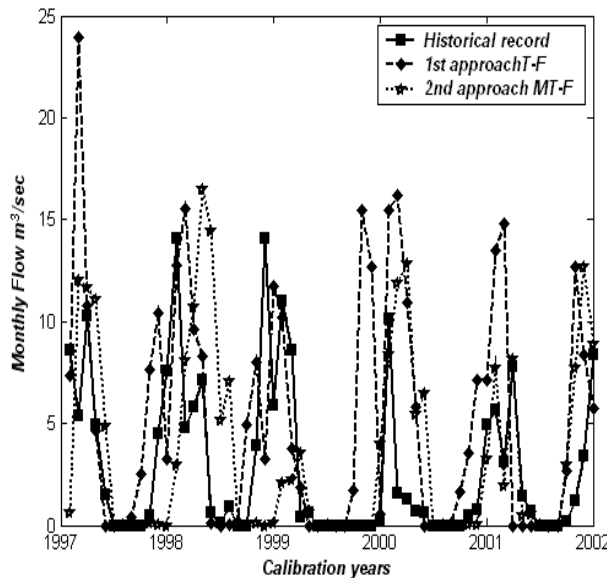


Figure (9) The comparison of Annual standard deviation of Khassa Chi flow for the synthetic data by two approaches

Figure(10) shows a comparison of two synthetic monthly flows by the two approaches with the historical one for the calibration years .

Figure(11) describes the Frequency distributions of the historical data and the two synthetic monthly flow by the two used approaches .



Figure(10) The comparison between the synthetic data for the two approaches with the historical data for calibration years

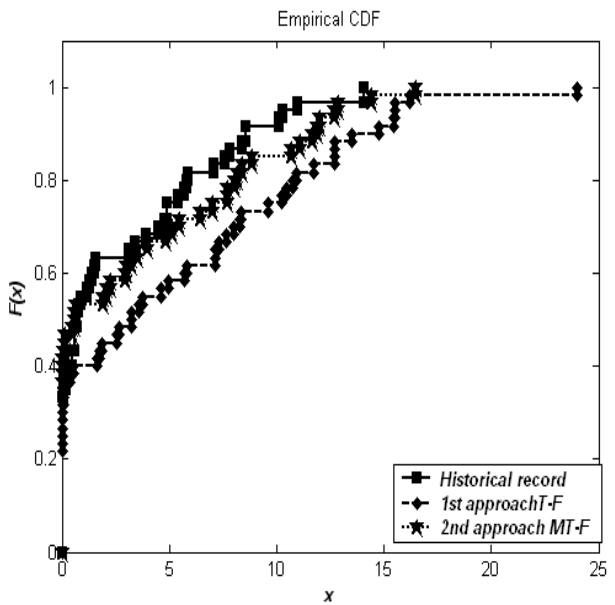


Figure (11) Cumulative distribution function comparison between the two approaches with the historical data



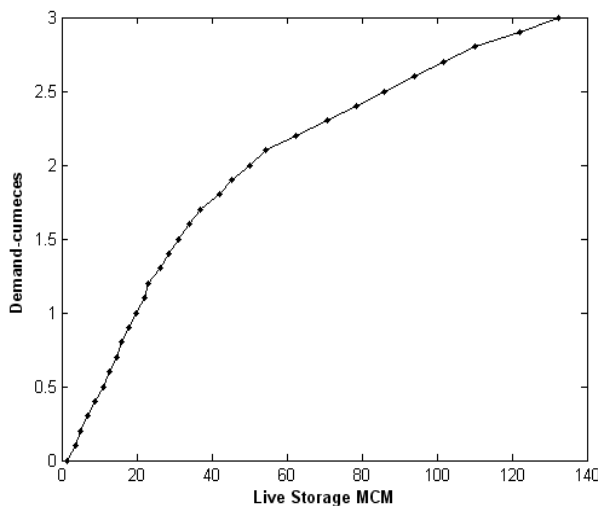
According to the above Figures in addition to some statistical tests which were done and shown in table(1) below to the generated flows

Table(1) The Statistical comparison of historical record and the synthetic values for the two approaches.

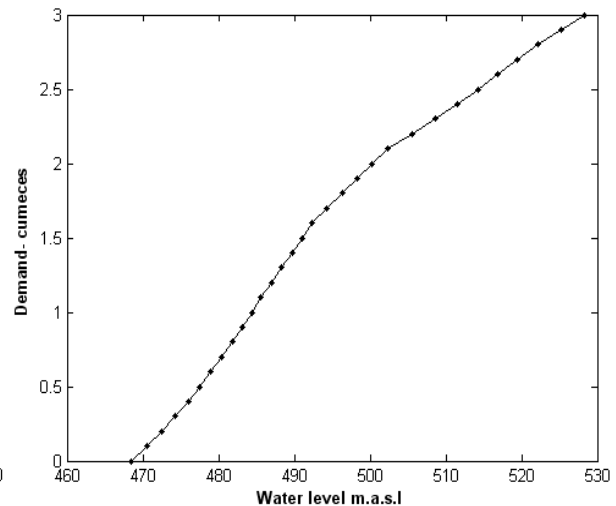
Tested Series Description	1st Approach T-F		2 <sup>nd</sup> Approach MT-F	
	t-calculated	F-calculated	t-calculated	F-calculated
Monthly mean values	-0.054	0.9247	0.0065	1.099
Monthly Standard deviation	0.1574	1.2156	0.0562	1.0464
Annual mean values	-2.0081	2.0432	0.0382	0.6784
Annual Standard deviation values	-2.8392**	6.421**	-0.09	0.8493
The whole generated Flows	-0.5288	0.4322	-0.2122	0.6694

\*\* Failed values with respect to the critical values.

It was clear that the Thomas- Fiering T-F model could be very good tool to simulate the Khassa Chi river, and it could also reverse all the important historical statistical parameters, when it is used for generating future values, and when it is used in its transformed form. It is more capable in reversing the statistical parameters of the historical record, with approximate conformity with historical record of monthly flow. As was mentioned before the simulation of reservoir operation was done with corresponding to the generated series and the results are shown in Figure(12) which shows the expected khassa Chi reservoir live storage verses water demand and Figure ( 13) which shows the expected reservoir normal operation water level for different water demand.



Figure(12)Reservoir live storage Vs.Demand for Khassa Chi dam



Figure(13) Reservoir water level Vs.Demand for Khassa Chi dam.

**Conclusions:**

Following are some conclusions were concluded from this study:

- 1- Thomas-Fiering model was very suitable to simulate Khassa Chi stream
- 2- Thomas -Fiering model was able to generate a synthetic data for a required period which was similar to the original data.

- 3- The new approach of Thomas- Fiering model was more capable to reverse the monthly and annual standard deviation of the historical data and its frequency distribution.
- 4-The modified Thomas- Fiering model was more sensitive to reverse drought times or months, which enables its usage in more comprehensive researches that dealing with variables suffering from drought times .
- 5-Presistency component in Khassa Chi stream was too effective part and it was most descriptive property of this stream.
- 6-The live storage will be  $31 \times 10^6 \text{ m}^3$  and the capacity of Khassa Chi reservoir is  $=39.1 \times 10^6 \text{ m}^3$  and the normal operation level is  $=491.08 \text{ m}$  above sea level.

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