# Aggregate Production Planning Using Goals Programming

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## Abstract

**This paper investigates Aggregate Production Planning (APP) model in a multi-plant producing multi-product to satisfy portion of fully deterministic demand in several cities for short term planning horizon. A Preemptive Goal Programming (PGP) approach is proposed with different scenarios to solve the APP model with conflicting multi-objective functions in order to maximize the total net profit with limited investment (budget), limited storage space, production capacity, and resources of the company. The proposed PGP model is also used to minimize the total production, inventory, transportation and defective items costs with optimum transportation pattern. A model is optimality solved and validated for a small numeric example of production planning problem with the results of optimal solutions for different scenarios obtained using optimization software LINGO package.**

**Keywords: Aggregate production planning, Linear programming, Multi-objective criteria, Preemptive Goal Programming, Transportation .**

# التخطيط الشامل للإنتاج باستخدام برمجة الأهداف

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## الملخص

**تحقق في هذا البحث بناء نموذج للتخطيط الشامل لإنتاج أنواع من المنتجات في عدة مصانع لتحقيق جزء من الطلب المحدد من قبل عدد من المدن لفترة قصيرة الأمد . أستخدم في النموذج المقترح طريقة برمجة الأهداف بمختلف السيناريوهات في حل مسألة التخطيط الشامل للإنتاج بعدة دوال لأهداف متضاربة لغرض تحقيق أقصى الأرباح بموارد استثمار، مساحة خزن ، طاقة إنتاجية ، وموارد أخرى محدودة. أستخدم النموذج المقترح أيضا في أيجاد أقل ما يمكن من تكاليف ألإنتاج ، الخزن ، ألنقل بين المدن ، وتكاليف القطع التالفة، مع أيجاد الخطة المثلى للنقل. جرى تطبيق النموذج المقترح على أحد أمثلة التخطيط الشامل للإنتاج لبيان فاعلية النموذج وتم استخراج نتائج الحلول المثلى لسيناريوهات مختلفة باستخدام البرنامج الجاهز LINGO.**

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## 1. Introduction.

APP is a process which assist the manufacturer to balance capacity and demand in such away that costs are minimized over a short term planning horizon from approximately 3 to 18 months into the future. Given the demand for each period, the APP specifies the production level, workforce, inventory level, subcontracting, overtime production and other controllable variables for each period while minimizing relevant costs over the planning horizon [1]. There are many strategies the decision maker DM can cope with demand fluctuations and costs associated with APP problems such as:

- Varying the production rate by introducing overtime, subcontracting, change workforce by hiring / laying off workers.
- Accumulating inventory in a period of low demand, and then used it to fill demand during periods of high demand [2].

The DM may select one or more of available strategies to be used efficiently in planning the production with least total costs over the planning horizon as the main objective, or other objectives that can be considered as maximizing (net profit, utilization of the plant or equipments, ...etc ), or minimizing the changes in ( workforce, production rate, inventory investment ,…etc) [3]. Several techniques are available to find optimal solution of APP model such as, linear programming, linear decision rule, transporting method, and simulation method [4]. Although mathematical linear programming optimization technique is successfully utilized in solving management problems of single objective function subjected to linear constraints, but it has a major limitation where it is inadequate technique for problems of more than single objective function, hence Goal Programming (GP) is a suitable technique to solve models with multiple objective functions. In this technique, each objective function is considered as a goal and the technique seeks to minimize the deviations between the desired goals and the actual results to be obtained according to the assigned priorities [5].

Recently, several approaches has been proposed to deal with APP models. According to saad [6], and Nam and Logendran [7], the APP models may be classified into 5 categories: (1) Linear decision rule. (2) Linear programming and transportation method. (3) Management coefficient approach. (4) Simulation method and (5) Search decision rule. Shi and Haase [8] designed an APP with multi-goals, multi-capacity demand levels. Leung and chan [9] designed a preemptive goal programming model to maximize profit, minimize repairing costs and maximize machine utilization. Dhaeneas-Fipo [10], formulate an integrated production problem in multi-facility, multi-product, multi-period environment. The model is solved using CPLEX software. Mostefa Belmokaddem [11], presents APP model for iron manufactures. The fuzzy goal programming approach was applied to minimize total production, inventory, and rate of change in workforce costs. The model is solved by using LINGO computer package. Tien-Fu Liang and Hung-Wen Cheng [12], presents a two phase fuzzy goal programming method for solving multi-product and multi-time period model. The designed model attempts to minimize total costs of change in labor levels, matching capacity with limited warehouse spaces and available budget. LINDO software is used to solve the model. Stephen and Leung, Yue Wu [13] use goal programming to solve APP problem with multi-objectives using LINGO software package.

This paper develops APP model consists of multiple conflicting objective functions with different scenarios, used to solve multi-plant, multi-product production problem, where all finished products should be shipped to several cities to satisfying some portion of the fully deterministic demand over the planning horizon, under the constraints of available production capacities of the plants, safety stock levels, and storage capacity limitations of the plants, and for the warehouses of cities. The model proposed is under the limited investment (budget ) of the company.

Preemptive Goal Programming (PGP) technique is used to solve the model to minimize the total production, transportation, stock inventory and defective items costs, within a limited budget (investment) as the first priority, and to maximize the net profit over the planning horizon as the second priority.

This paper is organized as follows. In section 2 a mathematical formulation of the multi-objective model is described and introduced as a general APP model. In section 3, the model is implemented in a numeric example as APP problem with different scenarios, then it is converted to PGP model with two conflicting goals. Results are obtained and discussed by using LINGO computer package. Section 4, contains the conclusions and scopes of future research.

## 2. Multi –objectives linear programming model formulation.

The following steps are used in order to formulate the model: [13][14]

**1) Indices**

i = the index of product i = 1,2,…..I .
j = the index of plant j = 1,2,..…J .
k = the index of city k = 1,2, …K .
t = the index of period t = 1,2,….T .

**2) Decision variables**

dfijt = number of defective items of product i manufactured in plant j at period t (unit).
Iijt = inventory level of product i manufactured in plant j at end of period t (unit).
Qijt = the quantity of product i manufactured in plant j at period t (unit) **.**
Sijkt = the quantity of product i manufactured in plant j sold to city k at period t (unit).

**3) Parameters**

aij = part of capacity used for manufacturing one unit of product i in plant j (hr/unit).
Cijk = unit transportation cost of product i manufactured in plant j and transported to city k at period t ($/unit).
Dikt = demand of product i in city k at period t (units).
Dfcijt = unit defective cost of product i manufactured in plant j at period t ($/unit).
hijt = unit holding cost of product i manufactured in plant j at period t ($/unit/period).
Iij min= safety stock of product i in plant j (units).
Iij max= max. storage capacity of product i in plant j (units).
Oijt =unit production cost of product i manufactured in plant j at period t ($/unit).
Mcjt = available capacity of plant j at period t (units).
rijt = selling price of product i manufactured in plant j at period t ($/unit).
SCk = storage capacity of warehouse of city k (units).
γij = % of average defective units of product i manufactured in plant j.
ψjt = % of actual production capacity used of plant j at period t .
Ɵikt = % of demand that should be satisfied of product i in city k at period t.

**4) Objective functions**

Generally, in linear programming models of APP problems, the objective function may be one of the following types: [8][13]

- *First objective function* is to maximize the total revenue over the planning horizon period .

$$MAX\ Z1 = \sum_i \sum_j \sum_t rijt.\ Sijt \qquad i \in I,\ j \in J,\ t \in T \quad \ldots\ldots\ldots\ldots..(1)$$

- *Second objective function* is to minimize the total production, transportation, inventory and defective units costs.

$$MIN\ Z2 = \sum_i \sum_j \sum_t Oijt\ .Qijt + \sum_i \sum_j \sum_t \Big(\sum_{K \in k} Cijkt\ .Sijk\Big) + \sum_i \sum_j \sum_t hijt\ .\ Iijt$$

$$+ \sum_i \sum_j \sum_t Dfcijt\ .dfijt \qquad i \in I,\ j \in J,\ t \in T \quad \ldots\ldots\ \ldots\ldots(2)$$

- *Third objective function* is to maximize the ratio of total return over the total investment.

$$MAX\ Z3 = Z1 / Z2 \quad \ldots\ldots\ldots\ldots\ldots..(3)$$

- *Fourth objective function* is to maximize the net profit.

$$MAX\ Z4 = Z1 - Z2 \quad \ldots\ldots\ldots\ldots\ldots..(4)$$

**5) Subjected to constraints**

- Production capacity constraint:

$$\sum_{i \in I} a\ ij\ .Qijt \le Mcjt \qquad j \in J,\ t \in T \quad \ldots\ldots\ldots\ldots\ldots(5)$$

- Inventory level constraints at each plant:

$$\sum_{j \in J} (\ Iij\ t\text{-}1 + (1\text{-}\ \gamma\ ij)\ \ Qijt - \sum_{k \in K} Sijkt\ ) = Iijt \qquad i \in I,\ t \in T \quad \ldots\ldots\ldots\ldots(6)$$

The inventory level in each plant j for each product i at any period t satisfy the safety stock, and not exceed the storage capacity.

$$I\ ijt \ge I\ ij\ min \qquad i \in I,\ t \in T,\ j \in J \quad \ldots\ldots\ldots\ldots(7)$$

$$I\ ijt \le I\ ij\ max$$

- Demand constraint at each city:

$$\sum Sijkt \ge \Theta ikt.\ Dikt \qquad i \in I,\ t \in T,\ j \in J \ \ldots\ldots\ldots\ldots(8)$$

- Nonnegative constraints:

$$Qijt,\ Sijkt,\ Iijt,\ df\ ijt \ \ge\ 0 \qquad i \in I,\ t \in T,\ j \in J \ \ldots\ldots\ \ldots..(9)$$

## 3. Model implementation.

**Numerical example**

XYZ is a small company experiencing in manufacturing two products GX, and GY in two plants namely plant A and plant B, with different capacities and located in different far locations. Each product can be manufactured in any of the two plants and shipped to different three cities namely (city 1, city 2, and city 3) to satisfy percentage of the requirement of the deterministic demand of the two products for the next six months horizon period. The following are the available information for the problem:

**1) The capacity**

- Each plant operates 8 hr/day as regular time with one shift, fixed number of workers, no overtime, no subcontract is allowed .
- Production rate of the two products in each plant is assumed constant during the planning periods, with different in available capacity of the two plants.

- There is initial stock of the two products at the start of planning period which represents the minimum safety stock in each plant.
- Factors such as planned maintenance, average breakdown times of machines, number of special holidays during the planning horizon are calculated for each plant from past data by the managers, and the net is given as percentage of actual capacity.

**2) The demand**

The company must satisfy at least 80 % of deterministic demand of the product GX and at least 75 % of product GY during the planning horizon period for all cities. Any amount produced of the two products can be accepted while it is within the storage capacity of the warehouses of cities .

**3) The Cost parameters**

- Fixed cost of the plants is not included in the model.
- The total production costs is estimated as the sum of machining, labor, raw material, maintenance and other overheads costs required in the production .
- The % defective items produced in each plant is estimated from the past data with its costs . These items can be reused after repairing.

Input data for our example is summarized in Table 1 through Table 3 .

**Scenarios of the management**

The aim of this study is to find an optimal production and transportation plan to satisfy the requirement of portion of demand of the two products GX and GY in the three cities over the next six months period. To achieve this optimal plan, DM discusses the following two scenarios:

**1) Scenario one**

The APP problem is solved as LP model using LINGO computer software package with total of (145) variables and (140) constraints. DM in each run selects one of the objective functions of the model (Z1, Z2, Z3 and Z4) which are listed in the expressions (1 to 4), subjected to the constraints in expressions (5 to 9). Results of optimal solution of APP based on given information by the company are listed in Table 4**.** Table 5. represents the optimal transportation patterns. The inventory levels are at minimum safety stock values in optimal solutions. Designing the model with this structure, and obtaining a different optimal solutions with different objective functions gives a flexibility and better decision support for the DM especially in case of limited resources like investment (budget) level. A DM can view the options available and choose a suitable strategy concerning the proposed APP model of the company.

The following are some important notes about the results listed in Table 4 and Table 5:

- All the demand for the two products GX and GY during the planning horizon are met in the model in any one of the four objective functions (Z1, Z2, Z3 and Z4).
- The optimal solutions of Z1 and Z4, are identical, so Z1 dominate Z4. The maximum investment (budget) required is $306,922 during the planning horizon with net profit $ 152,636 . This profit considered the maximum level which can be achieved with the available resources (capacities) of the two plants. The total production of GX product is (12080 units) which produced in plant A, while for

GY product is (11482 units) which produced in plant B, with fully utilization of actual available capacities.

- The objective function Z3 can be the best choice in case the level of investment available can not exceed $284,737 for the planning horizon. The optimal solution can yields a net profit of $145,154 with total of (12830 units) of GX product (750 units in plant B and others in plant A), and total of (9478 units) of product GY produced in plant B. The used capacity is full in plant A, while it is under the maximum available capacity in plant B in some periods.

Table (1) Demand for the two products ***GX, GY*** in six months and the maximum warehouse capacity of the two products in the three cities.

| Months | | Jan | Feb | March | April | May | June | Warehouse Max.capacity Units |
|---|---|---|---|---|---|---|---|---|
| Working days | | 26 | 25 | 27 | 26 | 27 | 25 | |
| City 1 | *GX* | 820 | 700 | 750 | 810 | 850 | 680 | 1500 |
| | *GY* | 740 | 760 | 830 | 700 | 820 | 880 | |
| City 2 | *GX* | 750 | 610 | 730 | 670 | 625 | 740 | 1300 |
| | *GY* | 660 | 580 | 520 | 630 | 670 | 690 | |
| City 3 | *GX* | 620 | 660 | 600 | 670 | 600 | 710 | 1400 |
| | *GY* | 680 | 630 | 610 | 635 | 660 | 690 | |

Table(2) Transportation cost

| | Transportation costs $/unit | | |
|---|---|---|---|
| | City 1 | City 2 | City 3 |
| Plant *A* | 0.50 | 0.35 | 0.55 |
| Plant *B* | 0.40 | 0.45 | 0.35 |

Table (3) Information available about the two plants

| Plant | Actual Capacity % | Holding Cost $/ unit / period | Product type | Selling Price $ / unit | Average Defective Items as % | Defective Item Cost $ / unit | Production Cost $ / unit | Production Time Unit/ hr | Safety Stock unit | Max. Storage Capacity unit |
|---|---|---|---|---|---|---|---|---|---|---|
| A | 88 % | 0.20 | *GX* | 18 | 2.5 % | 5 | 10 | 11 | 200 | 1000 |
| | | | *GY* | 22 | 3.0 % | 7 | 16 | 8 | 130 | 1000 |
| B | 92 % | 0.25 | *GX* | 18 | 3.5 % | 6 | 11 | 9 | 150 | 900 |
| | | | *GY* | 22 | 2.0 % | 8 | 15 | 10 | 100 | 900 |

Table (4) optimal solutions of APP model for the next six months planning periods

| Objective function | Optimum value $ | Total Revenue $ | TotalCost $ | Net profit $ | Total production (units) Plant A | | Total production (units) Plant B | |
|---|---|---|---|---|---|---|---|---|
| | | | | | *GX* | *GY* | *GX* | *GY* |
| Z1 | 459,558 | 459,558 | 306,922 | 152,636 | 12080 | 0 | 0 | 11482 |
| Z2 | 257,315 | 385,720 | 257, 315 | 128,405 | 10334 | 0 | 0 | 9478 |
| Z3 | 1. 509 | 429,892 | 284,737 | 145,154 | 12080 | 0 | 750 | 9478 |
| Z4 | 152,636 | 459,558 | 306,922 | 152,636 | 12080 | 0 | 0 | 11482 |

Table (5) Optimal transportation patterns with different objective functions numbers listed are the units of products GX and GY produced in plants A and B and shipped to (city 1, city2 , city3) during the planning horizon

☒ Objective function ( **maximize total revenue** Z1) . *(city 1 , city 2 , city 3)*

| | Product | Jan | Feb | March | April | May | June | Total |
|---|---|---|---|---|---|---|---|---|
| Plant A | GX | 656,794,512 | 604,488,795 | 639,910 ,490 | 648,658, 657 | 885,500,654 | 728,592,568 | 11778 |
| | GY | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0 |
| Used ψ | | 88 % | 88 % | 88 % | 88 % | 88 % | 88 % | |
| Plant B | GX | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0 |
| | GY | 844,505,526 | 895,435,472 | 861,390,696 | 852,547,477 | 615,800 ,532 | 696,554,554 | 11251 |
| Used ψ | | 92 % | 92 % | 92 % | 92 % | 92 % | 92 % | |

The total defective items are GX (302) , and GY ( 231) units.

☒ Objective function ( **minimize total costs** Z2 ) **.** *(city 1 , city 2 , city 3)*

| | Product | Jan | Feb | March | April | May | June | Total |
|---|---|---|---|---|---|---|---|---|
| Plant A | GX | 656,600,496 | 560,488,528 | 600,584,480 | 648,536,536 | 680,500,480 | 544,592,568 | 10076 |
| | GY | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0 |
| Used ψ | | 78.54 % | 73.473 % | 71.83 % | 77.10 % | 71.66 % | 79.44 % | |
| Plant B | GX | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0 |
| | GY | 555,495,510 | 570,435,472 | 622,390,457 | 525,472,476 | 615,502,495 | 660,517,517 | 9285 |
| Used ψ | | 76.53 % | 75.38 % | 69.44 % | 72.30 % | 76.18 % | 86.48 % | |

The total defective items are GX (258) , and GY (193) units.

☒ Objective function **( maximize the ratio of total return over total investment** Z3). *(city 1, city 2, city 3)*

| | Product | Jan | Feb | March | April | May | June | Total |
|---|---|---|---|---|---|---|---|---|
| Plant A | GX | 665,805,492 | 783,865,239 | 652,910 ,476 | 955,827,180 | 764,797,476 | 632,782,472 | 11772 |
| | GY | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0 |
| Used ψ | | 88 % | 88 % | 88 % | 88 % | 88 % | 88 % | |
| Plant B | GX | 0,0,0 | 0,0,300 | 0,0,0 | 0,0,450 | 0,0,0 | 0,0,0 | 750 |
| | GY | 555,495,510 | 570,435,472 | 622,390,457 | 525,472,476 | 615,502,495 | 660,517,517 | 9285 |
| Used ψ | | 76.73 % | 92 % | 69.64 % | 92 % | 76.37 % | 92 % | |

The total defective items are GX (308) , and GY (193) units.

☒ Objective function ( **maximize the net profit** Z4 ). *(city 1 , city 2 , city 3)*

| plant | Product | Jan | Feb | March | April | May | June | Total |
|---|---|---|---|---|---|---|---|---|
| Plant A | GX | 662,805,496 | 560,800,528 | 648,910,480 | 648,779,536 | 761,798,480 | 544,776,568 | 11778 |
| | GY | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0 |
| Used ψ | | 88 % | 88 % | 88 % | 88 % | 88 % | 88 % | |
| Plant B | GX | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0 |
| | GY | 555,495,825 | 570,435,798 | 637,390,920 | 539,473,864 | 615,503,829 | 660,518,625 | 11251 |
| Used ψ | | 92 % | 92 % | 92 % | 92 % | 92 % | 92 % | |

The total defective items are GX (302) , and GY (231) units.

The DM looks to the solutions of the problem with different strategies according to optimal solutions obtained with different objective functions (Z1, Z2, Z3 and Z4), and he makes trade-offs among these objectives. The selection of any strategy depends on his evaluation of the situation of the production requirements, like (available capacities, required demand, amount of investment level available …etc). Finally, the DM needs to choose only one preferable strategy that fits his own system of production.

**2) Goal programming approach GP**

Decision makers are usually faced with problems where they have to deal with many conflicting objectives such as maximizing the total revenue, minimizing the total relevant costs, and maximizing the utilization of plant or equipments. DM needs to optimize these conflicting objectives. GP is one of the most widely used, powerful and flexible technique used in such decision analysis. This technique is a special type of linear programming LP model developed by Charnes and Cooper In (1961). In LP models only one objective function is to be optimized subjected to several constraints but GP technique deals with many objectives at the same time and tries to work them together. GP cannot satisfy all objectives together, it tries to achieve all objectives while taking their priorities into account. In a GP model, constraints are turned in to goals and the objective is to minimize both positive and negative deviations from the goals [5].

A commonly used generalized GP model is expressed as follows:

$$MIN\ Z = \sum Pi\ (di^{-} + di^{+}) \qquad i=1,2,\ldots m$$

Subjected to :

$$\sum (a\,ij\ X\,ij) + di^{-} - di^{+} = bi \qquad i= 1,2,\ldots m, \quad j = 1, 2,\ldots n$$

Where :

$X\,ij$, $di^{-}$, $di^{+} \geq 0$ are decision variables.

bi is the goal i . And $di^{-}$, $di^{+}$ are deviations variables from the desired goals .

aij is the technological coefficient of the jth decision variable Xj in goal i.

Pi is the preemptive priority level assigned to each goal in ranked order .

P1 >>> P2 >> ………. Pm . This is assumed by DM. [9][13]

In non-preemptive GP model, equal weights are assigned to the deviations from the target values. In preemptive PGP model, priorities are assigned to each of the defined goal. The most desirable objective is given the highest priority, and the least desirable objective is given the smallest priority. The goals are worked in the order of priority and satisfied fully without disturbing the previous goals . [14] [15][16].

**3) Scenario two**

The management of XYZ company believes that to invest more than $ 270,000 as total production and transportation costs for the next six months planning horizon is extremely difficult, so a very high priority should be placed to avoid the increase in this capital investment above this level. Also, another goal is considered to exceed a net profit of $150,000 as a second priority. The objective is to minimize the sum of all deviations from desired conflicting goals .

The mathematical model of the PGP with these goals can be expressed as : [5] [13].

$$MIN\ Z = P1.d1^{+} + P2.\ d2^{-} \qquad \ldots\ldots\ \ldots\ldots(10)$$

Subjected to :

- Total cost goal (investment):

$$\sum_i\sum_j\sum_t Oijt.Qijt + \sum_i\sum_j\sum_t \Big(\sum_{K\in k} Cijkt\ .Sijkt\Big) + \sum_i\sum_j\sum_t hijt\ .Iijt + \sum_i\sum_j\sum_t dfcijt\ .Df\ ijt$$

$$+ d1^{-} - d1^{+} = \mathbf{270{,}000} \qquad i\epsilon I,\ j\epsilon J,\ t\epsilon T \quad \ldots\ldots\ldots\ldots(11)$$

- Net profit goal :

$$\sum_i\sum_j\sum_t rit \,.\, Sijt - \sum_i\sum_j\sum_t Oijt \,.Qijt - \sum_i\sum_j\sum_t \left(\sum_{K\epsilon k} Cijkt \,.\, Sijkt\right) - \sum_i\sum_j\sum_t hijt \,.\, Iijt$$

$$- \sum_i\sum_j\sum_t dfcijt \,.Dfijt + d2^{-} - d2^{+} = \mathbf{150{,}000} \qquad i\epsilon I ,\ j\epsilon J ,\ t\epsilon T \quad \ldots\ldots\ldots\ldots..(12)$$

- Non negative constraints :

$$di^{-} \ ,\ di^{+} \geq 0 \qquad\qquad i = 1,2 \ \text{goals} .$$

Other constraints of the APP model listed in expressions (5 to 9) remain without any changes. These constraints are concerning as technical limitation of the APP model. The priorities of the first and second goals are estimated by the top management of the company. The number (P1= 1000) represents the priority of the total cost goal, and number (P2=100 ) to the net profit goal.

The PGP model of APP problem is solved using LINGO computer software package with total of (150) variables and (143) constraints. Results of optimal solution are listed in Table 6. Table 7 represents the optimal transportation pattern. The inventory levels are at minimum safety stock values in the optimal solution. These tables reveal that the management can achieve its first goal, and production and transportation plan now can be applied within the limited capital investment of $270,000 as total ( production, inventory, transportation, and defective items ) costs for the next six months period, satisfying all the technical limitation of the plants. Table 6 shows that profit goal of $150,000 cannot be achieved within the limited capital investment, the net profit is underachieved by (***$ 13,328***). Fig. 1 and Fig. 2 describes that the total demand of each products GX and GY at each period is greater than the amount supplied by the two plants (as assumed by the model proposed). This is illustrating the limited actual capacity of the two plants which can not match the required demand of the two products with the limited investment level of the company. To mach this demand the company must increase the investment by purchasing new equipments, make overhaul maintenance to the machines used in production, work as overtime, propose development of the current production system, that is mean additional investment must be prepared by the company.

Table (6) Optimal solution of PGP model for the next six months planning period.

| Objective function | Total Revenue $ | Total Cost $ | Net Profit $ | Total production (units) Plant A | | Total production (units) Plant B | |
|---|---|---|---|---|---|---|---|
| | | | | GX | GY | GX | GY |
| **Z** | 406,672 | **270,000** | **136,672** | 1528 | 0 | 0 | 9478 |

Table(7) Optimal transportation patterns in PGP model. *(city 1 , city 2 , city 3)*

| plant | Product | Jan | Feb | March | April | May | June | Total |
|---|---|---|---|---|---|---|---|---|
| A | GX | 656,721,496 | 560,735,528 | 600,858,480 | 648,707,536 | 680,741,480 | 544,699,568 | 11525 |
| | GY | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0 |
| | Usedψ | 87.73 % | 78.14 % | 85.90 % | 88 % | 76.24 % | 88 % | |
| B | GX | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0,0,0 | 0 |
| | GY | 555,495,510 | 570,435,472 | 622,390,457 | 525,472,476 | 615,502,495 | 660,517,517 | 9474 |
| | Usedψ | 76.53 % | 75.38 % | 69.44 % | 72.30 % | 76.18 % | 86.48 % | |

The total defective items are GX (288) ,and GY (189) units.

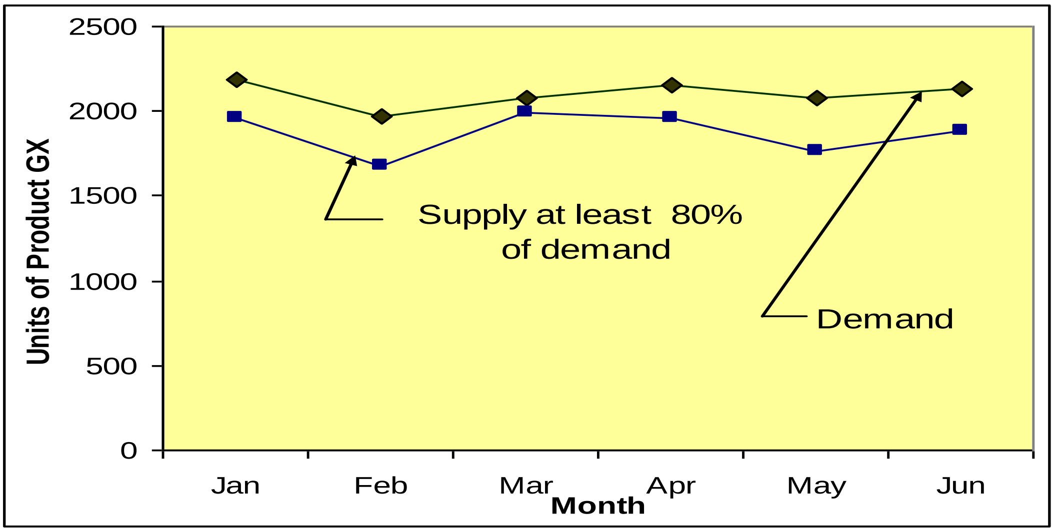


Fig. 1 Supply – demand curve of product GX.(results of PGP model)

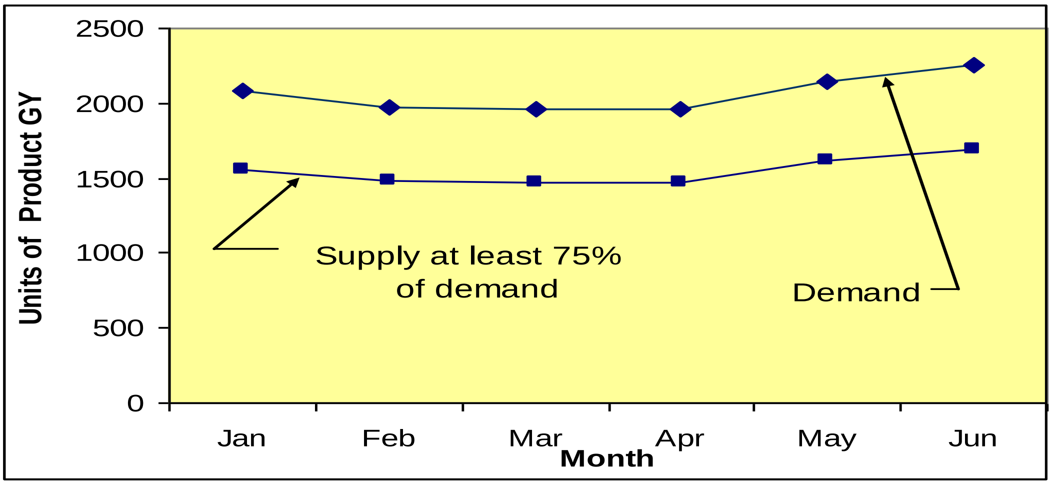


Fig. 2 supply-Demand demand curve of product GY.(results of PGP model)

**Other results of the model**

In such production planning system, the DM needs to evaluate various strategies and choose the one with highest return within the limited budget, satisfying all technical constraints of the proposed PGP model . The best selected strategy can be changed according to the ability of the management in increasing the investment. Fig.3 presents results of running the PGP model using different levels of investment during the planning periods. In each run the total cost goal (expression 11) is changed according to the certain investment level. The results indicates that the minimum investment required in the production system is $257,315 with net profit $128,405 , while the maximum investment is $306,922 with net profit $152,636 , so **the range of investment required for this company is from ($257,315 to $306,922), while the range of net profit is from ($128,405 to $152,636).** Any value more than this maximum investment will not make any improvement in the net profit since the two plants in this case working with full available capacity. This approach gives some advantages :-

- Analysis of several strategies aids the DM in selecting the suitable strategy at certain amount of investment and limited capacities.

- Comparison between different strategies of proposed PGP model reduces dependency of the human subjective decision.

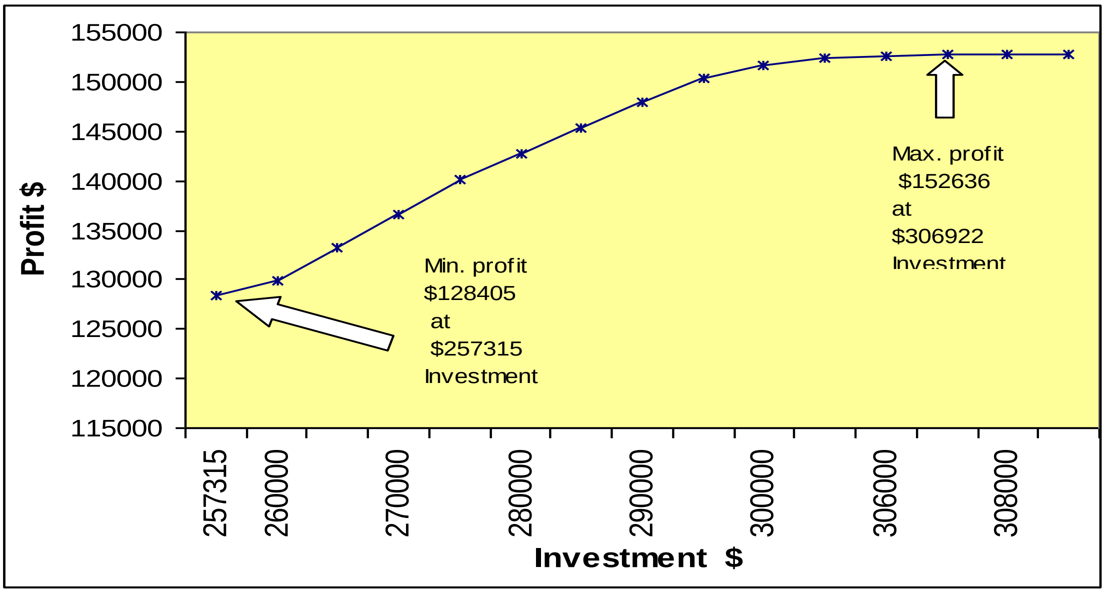


Fig. 3. Results of PGP model at different investment levels

## 4. Conclusions

This paper presents PGP approach for solving APP problems with multiple-objective goals and limited budget to satisfy portion of deterministic demand of multi-products in multi-time periods. The major goal of the proposed model is to maximize the net profit within the limited investment of the company with multi-plants and limited production capacities, storage spaces, and resources. The model also seeks to obtain optimum transportation pattern which should be applied to transport the finished products from the plants to three different locations. LINGO computer package is used to run the PGP model in simple numeric example which is used to demonstrate the effectiveness and flexibility of the propose model with regard to help the DM in development production plans based on different scenarios. Results obtained can assist a DM to choose a best strategy among various strategies the model presents. Based on the analysis of the results of running the propose PGP model, the following conclusions are drawn :

- PGP technique is simple and suitable tool for multiple conflicting objective goals, and it can be used in different fields of engineering applications.
- The model can be extended to any number of objectives by simply introduce the new goal as constraint, and the new objective is to minimize the deviations from the desired goal .
- The model support managerial decisions to develop APP problems particularly if resources budget are limited .
- The illustrated example is sufficient to lay a strong foundation on which a decision maker can formulate additional applications to large scale APP decisions .
- Encouraging researchers to explore the use of such models to be applied in real world situation with stochastic demand and capacities instead of deterministic variables.

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